Why Money Trickles Up
– Companies, Commodities and Macroeconomics

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0.0 Abstract

This paper combines ideas from classical economics and modern finance with Lotka-Volterra models, and also the general Lotka-Volterra models of Levy & Solomon to provide straightforward explanations of a number of economic phenomena.

Using a simple and realistic economic formulation, a model of corporate earnings is constructed that produces a power law distribution of company size by capitalisation.

A second model is produced to model the prices of commodities such as copper. Including a delay to capital installation; normal for capital intensive industries, produces the typical cycle of short-term spikes and collapses seen in commodity prices.

The third model combines previous ideas to produce a simple Lotka-Volterra macroeconomic model. This basic model generates endogenous boom and bust business cycles of the sort described by Minsky and Austrian economists.

From this model an exact formula for the Bowley ratio; the ratio of returns to labour to total returns, is derived. This formula is also derived trivially algebraically.

This derivation is extended to a model including debt, and it suggests that excessive debt can be economically dangerous and also directly increases income inequality.

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0.2 Introduction

This paper is a condensed extract from the full paper 'Why Money Trickles Up' which is available at econodynamics.org. The paper below is confined to the modelling structure and modeling outputs for the companies model, commodities model and the macroeconomic model. Background explanation and justification of assumptions has been kept to a minimum. To follow the modelling approach in this paper it is assumed that readers have already read sections 1.2 and 1.3 of 'Why Money Trickles Up – Wealth and Income Distributions'.

The paper introduces some mathematical and simulation models that use basic economic variables to explain the source of macroeconomic business cycles, including bubble and crash behaviour. The macroeconomic models are interesting, as even in these initial simple models, they give outcomes that accord closely with the qualitative descriptions of business and credit cycles in the work of Minsky and the Austrian school of economics.

The agents in the company models are identical, and simple in their behaviour. Because of this the agents had no utility or behavioural functions of the sort commonly used in agent-based economic modelling. As such the models had no initial underlying references to neoclassical economics.

As the modelling progressed, somewhat to my surprise, and, in fact to my embarrassment, it became clear that the models were modelling the economics of the classical economists; the economics of Smith, Ricardo, Marx, von Neumann (unmodified) and Sraffa. A key common factor between the companies, commodity and macroeconomic models is a clear differentiation between a fundamental value of capital, based on productive assets, and a financial value of capital, based on estimated future revenue streams. In all three models it is assumed that there can be differences between these two values.

Despite the rejection of neoclassicism, the models work. Classical economics works.

Where the classical economists were undoubtedly wrong was in their belief in the labour theory of value. They were however absolutely correct in the belief that value was intrinsic, and embodied in the goods bought, sold and stored as units of wealth. Once intrinsic wealth, and so the conservation of wealth is recast and accepted, building economic models becomes surprisingly easy.

Finally I would like to note that Ian Wright, Makoto Nirei & Wataru Souma have produced work on similar lines to my own, the parallels between their work and my own is discussed in section 12. I would also like to note that the general approach for the macroeconomic models in section 4 were partly inspired by the work of Steve Keen, though the models themselves grew straight out of my company and commodity models; and ultimately out of my income models.

More importantly, not a word of this paper would have been written without the work of Levy & Solomon and their GLV models. Manipulation of the GLV is beyond my mathematical ability. Although Levy & Solomon's economic explanations are naïve, their gut feeling of the applicability of the GLV to economics in particular, and complex systems in general, was correct. I believe their work is of profound general importance.

0.3 Structure of the Paper

Sections 2.1 to 2.4 go through the background, creation and discussion of a model that creates power law distributions in company sizes.

Sections 3.1 to 3.4 use ideas from section 2, and also the consequences of the delays inherent in installing physical capital, to generate the cyclical spiking behaviour typical of commodity prices.

Sections 4.1 to 4.4 combine the ideas from previous sections to provide a basic macroeconomic model of a full, isolated economy. It is demonstrated that even a very basic model can endogenously generate cyclical boom and bust business cycles of the sort described by Minsky and Austrian economists.

In section 4.5 it is demonstrated that an exact formulation for the Bowley ratio; the ratio of returns to labour to total returns, can easily be derived from the basic macroeconomic model above, or indeed from first principles in a few lines of basic algebra.

In section 4.6 the above modelling is extended into an economy with debt. From this a more complex, though still simple, formulation for the Bowley ratio is derived. This formulation suggests that excessive debt can be economically dangerous and also directly increases income inequality.

Brief conclusions are given in section 11.

Section 12 gives a history of the gestation of this paper and an opportunity to thank those that have assisted in its formation. Sections 15 and 16 give the references and figures respectively.

2. Companies Models

2.1 Companies Models - Background

Firms exist to protect their value-increasing property, their sources of negentropy. Firms buy goods that have well defined prices such as raw materials, components, electricity and labour.

They then use these inputs to go through a series of intermediate goods stages with, at best, indeterminate prices. To complete the process of production successfully, a company has to finish the goods to a well-defined point, where they can be easily priced in the market and sold to consumers or to other companies as intermediate goods. The company, with its plant, trained workforce, patents, designs and trademarks, exists to protect this wealth creation process.

In neo-classical economic theory the sizes of the companies should either be very small if entry to markets is easy, or very big and monopolistic; depending on the returns to scale. In fact it is well documented that company sizes, whether measured by number of employees or capitalisation follow well defined power law distributions [Gabaix 2009] or [Gaffeo et al 2003].

The model for companies in this paper builds on the income models introduced in section one of the full paper. The modelling looks at company sizes in terms of total capitalisation K of the companies. To extend these models, four basic assumptions are made.

Firstly, it is no longer defined that the valuation of the paper assets W matches the real capital of the company K. The short-term stock-market price W is allowed to vary significantly from the 'fundamental' value of a company's real capital K.

Secondly, it is assumed that shareholders are myopic, and judge expected company results simplistically on previous dividend returns.

Thirdly, it is assumed that managers of companies act to preserve the stability of dividend payouts, Fourthly, and more importantly it is assumed that managers act to preserve the capital of their companies.

Justifications for these assumptions are given in the full paper.

For the companies model, liquidity, and so company size and book to market values are assumed to be irrelevant. It is assumed that liquidity is constant and high throughout the modelling process.

In this model it is assumed that risk is identical, and in fact zero, for all companies in the model.

Given the above assumptions of zero risk and high liquidity; following Fama & French [Fama & French 1992], this leaves short term returns as the only factor that investors use to value companies.

So the present value of a company is given simply by:

$$\text{Present Value} = \frac{\text{Dividend}_1}{\text{r}}$$

Where r is the relevant market interest/profit rate; Dividend_1 is the latest dividend payment, and capital growth is ignored [Brealey et al 2008, chapter 5].

2.2 Companies Models - Modelling

Figure 2.2.1 here

Figure 2.2.1 above is a slightly modified version of figure 1.3.5.

We are now looking at the financial assets from a company point of view, and we are not interested in the individuals. There are N companies, from j=1 to j=N.

We have removed the assumption that K = W or that $k_j = w_j$.

So here we differentiate between the fundamental value of the real capital k_j formed of the firms buildings, plant, patents, etc and the market valuation of the company w_j. w_j represents the sum of the stock market value of paper share certificates held by the owners of the company.

(Note here that w_j is the total wealth represented by all the shares in company j held by various different individuals – w_j is not the same as w_i of section one.)

At the beginning of each simulation we start with $\Sigma k_j = K$ for all the companies, and also $\Sigma w_j = K$. Initially all the companies are the same size, and all are valued fairly by the stock market, with the fundamental value of each company equal to its market capitalisation.

It is assumed that each of the j companies has a standard rate of growth r_j. The average $\bar{r}_j$ will be 0.1, that is each company produces value roughly equal to 10% of its capital each year. So each of the companies is identically efficient in the use of their capital. However r_j is stochastic about this long-term average.

It is assumed that the market is not well informed about the fundamental value of individual companies. Following the research of Fama & French and others, it is assumed that investors simply use the average market rate of returns (0.1 or 10%) as their guide for valuing companies.

So the new market capitalisation w_j for each iteration of the model will simply be the last actual real returns $\pi_{j,t}$ divided by the long-term rate of returns.

$$\text{so:} \quad w_{j,t+1} = \frac{\pi_{j,t}}{r}$$

Then the expected returns for the next year will be the market capitalisation Wj multiplied by the average market rate of return.

$$\text{so:} \quad \hat{\pi}_{j,t+1} = w_{j,t} r$$

Which is an unnecessarily complicated way of saying that next year's expected returns will be the same as the previous years actual returns.

As in the previous models, we will assume that labour is fairly rewarded for the amount of added value that it is supplies. So L = e exactly, and both L and e can be ignored in the mathematical model.

The loop of the simulation was carried out as follows:

The amount of production is calculated by multiplying the capital of each company by the relevant production rate, so:

$$\text{production} = k_{j,t} r_{j,t}$$

After a round of production all of the companies will receive cash from purchasers of its manufactured goods. This cash value will represent the value added in the production process.

Each of the companies will have a value of expected returns ($\hat{\pi}_{j,t}$) based on its current market capitalisation.

In the simulations carried out actual payouts of profit π were varied by using different payout ratios.

The managers might pay out 90% of the earnings, retaining 10% of the extra value, so allowing a buffer to be built up against future problems, also to allow expansion of the company, empire building, etc. This extra value is added to the total capital.

If the managers only pay out 90% of the earnings, this is defined from now on as an 'payout ratio' of 90%. The model allows different payout ratios on the upside and downside. So managers may have an upside payout ratio of 90% and a downside payout ratio of 80%. This would mean that the management would pay out 90% of the earnings if earnings were greater than market expectations, but would only pay out 80% of earnings if earnings were less than market expectations.

For example in model 2B both the upside and downside payout ratios were 90%.

These actual payouts then give the market its new information for resetting the market value w_j of the various companies.

The capital k_j of each company is then recalculated as follows:

$$k_{j,t+1} = k_{j,t} + \text{production} - \text{actual\_returns}$$

This model assumes a stationary economy with a fixed total amount of capital K.

This capital can be bought and sold between different companies, as they are required to give earnings in requirements of market expectations.

All of the companies will receive cash from purchasers of its manufactured goods. This cash value will represent the value added in the production purpose.

Some companies will receive more cash than they are expected to payout, some will receive less.

It is assumed that the cash rich companies will purchase real capital off the cash poor companies, so allowing the cash rich to expand, and the cash poor to pay their earnings.

At each round of the modelling process, the sum of the capital is renormalised to the original K. This is because asymmetric retention of funds allows excess growth or decline for the whole economy.

2.3 Companies Models - Results

2.3.1 Model 2A Fully Stochastic on Production, No Capital Hoarding

Model 2A is the simplest model, so simple that it inevitably fails.

Firstly the model is completely stochastic. Each company produces output worth exactly 10% of its capital on a long-term average. However the value of 10% varies up and down stochastically according to a normal distribution.

In model 2A the payout ratio is set at 1. This means that the managers of the companies payout the full amount expected by the market. They do this no matter how well, or how badly the companies perform.

Figure 2.3.1.1 shows the full log-log distribution of all the (non-negative) companies. Figure 2.3.1.2 shows the power tail with the trend line fit for the power tail.

Figure 2.3.1.1 here

Figure 2.3.1.2 here

Companies that lose money, due to poor production, still pay out to market expectations, so they slowly drain their capital and lose it to other companies that have above average production. Because of this the model is not stable, and the distribution changes as the model progresses.

Despite this, it is noticeable that the model quickly generates a stable power tail with an exponent close to –1; close to the value seen in real life. The power tail remains stable from 10k to 50k iterations. Above 50k iterations the number of companies being eliminated (going negative) becomes very large and the transfer of capital to the larger companies starts to change the exponent of the power tail.

The important thing to note here is that a very simple model, using the standard valuation system of capitalism, quickly generates a power tail of companies of vastly different sizes. In the 50k, run power tail companies vary in their capital between 80k units and 80,000k units. But all the companies are absolutely identical in their earning ability, effectively the companies have identical managements making identical products with identical inputs. The differentiation in size has only occurred through the stochastic forces of chance.

2.3.2 Model 2B Fully Stochastic on Production, Capital Hoarding

Model 2B is identical to model 2A in that the companies are identical in average earnings, but these earnings vary stochastically from model to model.

Model 2B is different in that the payout ratios were changed in an attempt to create a stable model. Unfortunately this proved difficult. The only values that prevented 'washout' of smaller companies were payout ratios of 0.9 on both the upside and downside. Initial investigations suggest that this is related to the production rate of 0.1.

The results are shown in figures 2.3.2.1 and 2.3.2.2.

Figure 2.3.2.1 here

Figure 2.3.2.2 here

Unfortunately this model is a bit too stable. Although it shows a very clear power law, still with identical companies, the exponent of the power law is very different to that seen in the real world.

It appears that the retention is too great and is forcing a high minimum value for companies, so preventing the formation of the power tails with slopes seen in model 1A.

2.3.3 Model 2C Deterministic on Production, Capital Hoarding

In model 2C the production rates of the companies was set prior to running the model, and were again drawn from a normal distribution. So in this model some companies produced more than 10% all the way through the model, some produced less than 10% all the way through the model.

Note that model 2C is not stochastic, it is deterministic.

In this model some companies are more efficient than others with their use of capital.

Again the payout ratios were adjusted to prevent elimination of companies from the bottom of the distribution. It was found that any downside payout ratio of less than 0.5 or so prevented this washout.

Figures 2.3.3.1 and 2.3.3.2 below are for a downside payout ratio of 0.5 and an upside payout ratio of 0.9.

Figure 2.3.3.1 here

Figure 2.3.3.2 here

Intriguingly the power law exponent of –0.68 is close to the value of –1 seen in real life. However the fit is poor, and it turns out that the value of the exponent is highly sensitive to the value of the upside payout ratio and can change to high tens or low decimals for small changes in the upside payout ratio. Initial modelling suggests that the value of 0.9 is closely related to the production ratio of 0.1. As the production ratio is changed, an upside payout ratio of one minus the production ratio gives a power tail close to one.

Again, the important thing to note is that relatively small changes in relative efficiency of the companies produces a power tail with very large, multiple factors of ten, differences in size for the companies.

2.4 Companies Models - Discussion

As can be seen from the results, using a very simple combination of classical economics and dynamic statistical mechanics allows the building of simple models that give power law distributions for company sizes similar to those found in real life economies.

As with the income models it noticeable that there are many things that are not needed to produce such a model, these include:

- Economic growth
- Population changes
- Technology changes
- Different initial endowments (of capital)
- Shocks (exogenous or endogenous)
- Marginality
- Utility functions
- Production functions

The issue of marginality, utility, production functions will be returned to in a moment, before that I would like to discuss the roles of shocks, expectations and behaviouralism.

It is notable that the models do not include for exogenous shocks, which are often found in explanations of company size.

Models 2A and 2B are stochastic, and do therefore model minor endogenous shocks to productivity. These could be issues such as a variation in breakdown rates of machinery, management efficiency, etc from period to period. What is notable about models 2A and 2B is that the average productivity of all companies over the long term is identical; and yet a power law still results.

Model 2C is effectively deterministic. The initial productive efficiencies of the companies are determined prior to the simulation. The simulation then rapidly reaches an equilibrium with a power law distribution. There are no shocks in model 2C; external or internal.

Expectations and behaviouralism do enter into the model in two different ways, firstly with regard to the pricing of stocks, and secondly with regard to the retention of capital within companies.

In both cases these are very obvious forms of behaviour and are supported by economic research.

With regard to returns, the assumption is simply to take the pricing of financial assets as strictly based on their recent returns. This is in fact the "traditional" naïve neo-classical form of pricing capital and is supported by the research of Fama & French and other work discussed in section 2.1 above. This assumption that prices of assets are defined by simplistic projections of present earnings is also at the heart of Minsky's theories.

The assumptions on capital retention are more subjective than the assumptions on returns, and more arbitrary in the specific amounts of returns chosen, and is the weakest part of my company modelling. This is discussed in more detail below, when comparing with the work of Ian Wright.

What is important to note is that the above assumptions on expectations are the only assumptions needed. No detailed assumptions about the understanding of the economy, interest rates, growth, technology, etc are needed.

The only 'behaviourism' that we need to assume is that, firstly investors are deeply short sighted, and secondly that managers don't like sacking themselves.

It is clear from the models that neither utility nor marginality are relevant.

Much more importantly, the output distribution for the models is demonstrably not 'efficient' in the normal neo-classical usage.

To take models 2A and 2B as examples, capital is rapidly shifted between companies according to short-term results, and companies with equal long-term efficiencies end up being sized very differently. In a neo-classical version of model 2A of 2B, either one company would dominate, or all companies would be equally sized.

Model 2C is far more realistic, and much more interesting. It also shows how profoundly free markets fail to allocate capital effectively.

Model 2C has a range of production efficiencies. Some companies make better use of their capital than others.

In a neo-classical outcome (or indeed in the classical models of Smith, Ricardo, etc) the outcome of such a model should be crystal clear. The most efficient company should continually be rewarded with more capital until it ends up being a monopolist, owning all the capital in the economy. Despite the best efforts of managers to cling on to their capital, investors should continually remove their capital from all the less efficient companies until these companies have no capital left and go out of business.

This is not what happens.

In model 2C, and as Graham, Buffet and others have discovered, also in real life, poorly performing capital is simply written down.

Companies are allowed to retain some of their real, book value, capital K. But part of their financial wealth is written off. Once an under-performing company's financial wealth W is small enough to make the (poor) returns from the actual K equal to the normal market rate, then the company is allowed to continue under-performing, and under-utilising its capital, indefinitely.

So it is noticeable that moderately bad companies are only downgraded, they are not driven out of business as economic theory suggests they should be.

This represents an enormous misallocation of real capital.

In model 2C the top company has a capitalisation/capital ratio of 1.37, the bottom company has a capitalisation/capital ratio of 0.62. The bottom company is half as efficient as the top company, but once it has been written down, it is allowed to limp on inefficiently.

That this happens in real life is supported by the effective long term investing models of Benjamin Graham, Warren Buffet and others. In an economy such as model 2C above, the Graham/Buffet approach is straightforward.

Finding companies with under-valued physical assets is straightforward; you simply look at the book value of assets compared to the stock price.

Generally it is poorly performing human capital that has driven companies into under-performance. The quality of human capital is something that can change very quickly.

The Graham/Buffet approach uses various measures to identify increases in the efficiency of human capital. These include qualities such as paying down debt and good recent dividend history.

By this process, investors such as Graham and Buffet can identify companies that are undervalued, with under-performing capital, and that are also likely to move quickly to over-valuation.

The workings of the model above are straightforward, and similar to the other GLV models. The companies have a positive feedback loop which means that the more companies earn, the more capital they get.

There is also a negative feedback loop, so the bigger companies get the more income they have to pay to investors.

If these were the only two rules, then the most efficient company would grow explosively into a monopoly. A true power law distribution can not go down to zero, so to be stable, a power law always needs some other distribution to 'support' it. That is why power law distributions are normally 'tails' to other distributions. As Levy & Solomon make clear, there needs to be a 'reflective barrier' above zero.

The assumption of retention of capital assures a continuous, if minimal income to all companies, however small. This prevents collapse of the distribution to a single point, and allows the generation of the power tail distribution.

This is the weakest part of the model above, with factors 'selected' to ensure the distribution does not collapse. While these assumptions are somewhat contrived, the work of Wright shows that different, but similar assumptions are just as effective.

In the modelling of companies the models of Ian Wright are significantly different to, and significantly better than, my own. Wright allows for the extinguishing of companies as they become too small to trade, and the creation of new start-up companies as individuals become sufficiently wealthy to employ other individuals. This avoids the somewhat artificial 'capital hoarding' approach that is used in my own model, which maintains all companies as operational entities, however severe their losses.

In real life clearly both mechanisms operate, with bankruptcy and new company formation happening alongside poorly performing companies that limp on for years without giving good returns on their capital. A third mechanism of corporate takeover, divestment and splitting of companies also takes place. Detailed research would be needed to determine the relative importance of the different mechanisms. Personally I believe that Wright has identified the most important factor in new company formation and extinction.

The main point is that, as long as you have a means of supporting the base of the distribution, the basic pricing mechanisms of capitalism produce a power law tail as seen in reality.

3. Commodity models

The following is a brief model, mainly to introduce some concepts and demonstrate the importance of a dynamic modelling approach to markets.

This paper has taken a classical economics approach that assumes that all goods and services have a meaningful intrinsic value that ultimately relates through to basic concepts of entropy in physics and biology.

It is immediately obvious that the prices of some goods; land, housing, gold, artworks, cabbage-patch dolls, etc, show wild fluctuations in price that appear to contradict the assumptions of fundamental value in classical economics.

To investigate this further a simple dynamic model of a commodity market is constructed, largely following the lines of the previous company model.

3.1 Commodity Models - Background

The model aims to model the behaviour of mining or agricultural commodities such as copper, aluminium, nickel, platinum, coffee, tea, cocoa, sugar, etc.

Such commodities have wildly fluctuating prices, normally characterised by long periods of low prices punctuated by severe spikes. The figure 3.1.1 below for copper shows a typical example.

Figure 3.1.1 here

This pattern is also seen in other commodities such as oil or natural gas, land, housing, etc.

While it is believed that similar forces operate in the markets for oil and houses, these commodities are sufficiently important that they can in turn have large impacts on the economy as a whole.

For simplicity the model below chooses to model something like copper or sugar that can have large price spikes without having a significant effect on the economy as a whole.

Commodities such as copper and sugar share three important factors.

Firstly, in a stable economy demand for these things is quite stable and relatively insensitive to price.

Secondly these commodities are non-substitutable.

The third factor is that all the above commodities take a long time to increase their output by installing new capital. For commodities such as these, price signals take a long time to result in increased output. It is this delay that changes the problem from one of comparative statics to one of dynamics.

3.2 Commodity Models - Modelling

This model follows on from the companies model above, and is simple enough to be modelled in a spreadsheet. In this model one section of the economy is modelled as a single unit, so there is only a single set of equations running in the model.

For the sake of the argument, assume the commodity is copper.The production cost of copper is fixed and related directly to its inputs, a mix of energy, machines and various types of labour. We have assumed that the price of copper, even if it varies dramatically, has very little effect on the economy as a whole. This means that the prices of the inputs of energy, machines, labour and any other inputs vary negligibly with the price of copper.

So the cost of producing copper is a simple linear function of the amount of copper produced.

As with the companies and incomes models, the total amount produced is a fixed ratio of the capital installed. Taken together this means that the marginal price of extra copper is strictly zero.

The price of copper is a different matter. It is assumed that total demand for copper is almost constant with a 'normal' amount required in the market place. In this model 100 units of copper.

When this amount, or more, is available, copper companies charge the costs of production. Also they lower their output by closing down excess capacity, and drawing down their capital as in the companies model. This gives a base price for copper of 1.0 in this model.

If production drops below that required, then price increases very rapidly and demand is choked off very slowly, the demand is highly inelastic. Figure 3.2.1 below shows the price volume curve used in the model.

Figure 3.2.1 here

In this model, just as in the companies model, the standard market interest rate defines the expected returns, based on the previous market capitalisation w.

Again, as in the previous model, payouts are predicated on the expected returns using payout ratios, with companies hoarding capital or returning it to shareholders as appropriate.

When supply is low, and prices jump up, the mining companies find themselves with much higher receipts than costs. In these circumstances the excess cash is used to provide more capital.

As discussed above, this capital is added to the productive capital, but only after a lag of a number of iterations. This lag can be adjusted in the model from zero to ten cycles.

Once the new capital has been added after the lag in time, then production can be increased. Eventually this allows supply to meet demand and prices can drop again.

3.3 Commodity Models - Results

The results are fairly straightforward.

Figure 3.3.1 below gives the output for Model 3A, this shows the prices for copper with no lag on capital installation and payout factors of one; ie no capital hoarding.

Figure 3.3.1 here

Even with this very simple model the system is unstable and produces wide cyclical variations in prices The real price of copper, based on inputs, should be 1 unit; note that the system is only at its true input price for short periods of time. Left to itself the market charges an average price slightly over 50% of the input cost price. The extra 50% being caused by the cyclical over-production and destruction of capital, and consequent rent taking.

Figure 3.3.2 below for model 3B shows a capital lag of two periods, but still with a payout ratio of one.

Figure 3.3.2 here

This shows a pattern closer to reality; long periods at 'classical' prices are interrupted with intermittent spikes. Even in this simple model it is notable that the spikes have a variable pattern showing chaotic (not stochastic) behaviour. With this capital lag, the average price is raised to 1.7 times input cost, as the cycles of capital creation and destruction become more aggressive, and rent taking becomes larger.

3.4 Commodity Models - Discussion

The first point to note is that simple dynamic economic models can result in complex chaotic behaviour that mimics real life surprisingly well.

The behaviour is chaotic, not stochastic. The random changes are generated endogenously. There is no stochastic generator in this model.

This is a Lotka-Volterra model, not a General Lotka-Volterra model. This model is very similar to the lynx and hares model first discussed back in section 1.2, in fact it is closer to the Soay sheep and grass model. The build up of excess capital in the mining companies is analogous to the build up of excess sheep biomass on the island of Soay. The build up of capital is too much for the economy to support, as the build up of sheep is too much for the island to support. While the GLV models were stable, like many Lotka-Volterra models, the build up of capital in the commodity sector is inherently unstable. The problems are deep in the maths of the system. Blaming investors or speculators for misjudging their investments is as sensible as blaming the sheep for procreating.

Diminishing returns and marginality are conspicuous by their absence.

Diminishing returns are not needed for the model to work. Neither is marginality, and any costs associated with marginality are of an order smaller than those associated with dynamic effects.

Using classical economics within a dynamic framework works. It produces output prices that can be at substantial variance with input prices, and can vary substantially with time.

It should also be noted that the model does not average to the correct input prices even over the long term. The correct input prices are instead associated with the bottoms of the cycles, and are only touched for short periods of time.

Due to problems associated with the way assets are priced, the time taken to install capital, and (financial) capital hoarding by companies, the market is profoundly inefficient. Average prices are substantially higher than they would be if they had the opportunity to settle to long-term static equilibrium prices.

The form of this over-pricing is interesting. Above I referred to it as associated with capital appreciation and destruction, but the process is more subtle than this.

In a boom period, customers are substantially overcharged compared to the input costs. Extra capital is created, but the nominal capitalisation increases much faster than the real value of the capital installed. In short the companies become grossly overvalued. As a consequence they pay excessive dividends. In a boom most of the over-pricing passes straight through to shareholders as excess profits.

In the following crash, the company is still expected to match dividends at the market rate. It does so by drawing down capital to pay dividends.

Over the cycle as a whole customers are forced to overpay, with the payments transferred direct to excess profits. Allowing dynamic cycling of economic variables in this way allows large-scale rent-taking by the owners of resources.

4. Minsky goes Austrian à la Goodwin – Macroeconomic Models

4.1 Macroeconomic Models - Background

Although the GLV has not previously been used significantly in economics, some non-linear modelling work has been carried out at a macroeconomic level by Kalecki, Kaldor, Desai and others. Most notably Goodwin used the Lotka-Volterra predator-prey system to model a qualitative cycle described by Marx. Keen has extended the Goodwin model to model a Minskian business cycle [Keen 1995].

There is significant evidence to suggest that these cycles exist in real economies. Barbosa-Filho & Taylor [Barbosa-Filho Taylor 2006] have carried out a detailed study of business cycles in the US. Harvie [Harvie 2000] has carried out a similar study for ten OECD countries. In both cases the evidence is qualitatively strongly suggestive of cyclical changes in labour share of return and employment that match the patterns predicted by Goodwin. In both case though there are significant difficulties in fitting the data quantitatively.

In addition to the work above there have also been substantial qualitative studies of business cycles in other schools of non-orthodox economics.

In the Austrian school, it has long been proposed that the build up of excess capital has been a fundamental cause of business cycles, with the blame for this generally put on government mishandling of credit availability.

In parallel with this Minsky, coming primarily from the post-Keynesian school, but also following the work of Fisher, has also studied the build up of economic cycles, though with the blame being primarily placed with speculation and the unsustainable endogenous creation of debt.

The Austrian and Minskian models share significant common features, the most obvious being their beliefs that booms and busts are natural features of economics. Another, unfortunately, is their shared disdain for formal mathematical modelling.

In the modelling that follow a very simple macroeconomic model is built, that combines the Lotka-Volterra approach of Goodwin with the basic ideas of the Austrian / Minskian business cycles.

The main ingredients for this model, including many simplifications, are already available in the proceeding models above.

4.2 Macroeconomic Models - Modelling

The main assumptions of this model are as follows:

In line with classical economic theory, produced goods have real values, but market prices can vary from these values in short time periods due to insufficient or excess demand.

Consumption is a fixed proportion of consumers' perceived wealth, held in the form of paper assets, as in the income models.

Companies have real capital which can produce a fixed amount of output, and needs a proportional supply of labour, as in all the models above.

The price of paper wealth assets is defined by the preceding revenue stream; as in the myopic companies model above.

The management in companies can be capital preserving, as in the companies model above.

There can be delays in installing capital as seen in the commodities model above.

The price of labour is non-linear according to supply. That is real wage rates go up when there is a shortage of labour, and go down when there is a surplus of labour. Labour is a genuinely scarce resource.

It should be noted that, unlike the Goodwin models, both population and technology are fixed.

As with the commodities model, the economy as a whole is analysed, with the focus on time dependence.

The big new assumption in this model is that labour costs vary with employment and unemployment.

It is assumed that labour costs vary as a concave function of employment, ie labour costs will increase as the employment ratio increases, and will increase at an increasing rate.

Figure 4.2.1 here

In this model I have used a simple square law function, shown in figure 4.2.1 above. This is not a particularly realistic function, more realistically it should be asymptotic to the vertical on the right hand side as there is a realistic maximum somewhere around 6000 hours per year. However this basic function is sufficient for the needs of the model.

It is also worth noting, this is not an inflation Phillips curve. This curve is a simple supply-price Phillips curve for labour in real terms. In this model, prices of goods and labour both go up and down, just as they did in the commodities model, but they move around stable long-term values. The analogy is with the cyclical price changes seen in a Victorian economy with a gold standard. There is no long-term monetary inflation. For a pithy study of the misinterpretation of the Phillips curve see Hussman [Hussman 2011].

In these models, it is assumed that individuals always spend 40% of their income at all times, $\Omega = 0.4$.

It is possible that the consumption spending will exactly balance the amount of production capacity available in the companies, however this will not always be the case. It is also possible that there will be too much or too little capital available to match the consumption demand.

Looking firstly at the case of too little demand; if the 40% spending provides insufficient demand, then excess capital will be available and some of that capital will be unused. As a consequence of this there will also be a reduction in labour employed.

Also, following exactly the same logic as the companies models above, if companies create insufficient wealth to meet the payout targets set by their market capitalisation, then they will be obliged to convert some of their capital to wealth for payout.

It is also possible that there may be insufficient capital available. In these circumstances it is assumed that consumption is still maintained at the full 40% of current wealth, even though insufficient capital available, and so insufficient goods are produced. In this case the consumption funds available for purchasing are simply divided amongst the goods that are available to be purchased, so increasing the nominal market price of the goods above their long-term natural prices. Consequently this results in short-term consumer price inflation.

It is implicitly assumed that consumers judge value by price and continue to spend a fixed proportion of their wealth, even though they actually receive less real value for that wealth.

When this happens super-profits are then earned by the corporate sector. If employment and so wage levels are low, then the income retained by the companies is converted into new capital to allow the production of more commodities. In this manner, super-profits are converted into new capital and new production until supply rises to meet the new demand, and the prices of consumer goods then drop back to their 'natural' values based on input costs.

It is important to note that, in the company models, the total amount of capital was fixed; however in this macroeconomic model, the amounts of capital and labour employed can vary, though labour is still needed in a fixed proportion to capital used.

In this macroeconomic model the capital and labour are still used in a fixed ratio to give a given output.

The amount of capital can vary freely, in line with the demand of goods from consumers.

The total supply of labour is fixed however, with the amount of the labour pool employed varying in fixed proportion to the amount of capital. Labour costs vary non-linearly with the amount of labour employed, which means that labour costs vary non-linearly with the amount of capital employed. So returns to labour and capital can vary.

It is still assumed that the proportion of labour required to capital is fixed over the whole period of time being modelled. This means that there is no technological progress, and also that it is not possible to substitute capital for labour.

Each iteration of the model operates as follows:

The expected returns are defined as 10% of the current market capitalisation.

The consumption, and so the payments made for consumer goods are defined as 40% of total wealth.

If these payments are less than 20% of the available capital, then the amount of goods produced is equal to the value of the consumer payments.

If the payments for consumer goods are greater than 20% of the available capital, then the goods produced are equal to 20% of the total capital, ie, the maximum production possible is 0.2 times the capital K that is in existence.

The income accruing to labour is calculated, according to the amount of capital used, and so the proportion of labour employed, according to the square law.

The surplus revenue that the company generates is then the value of the consumer payments received, less the earnings income paid out.

The new value of the total real capital is then the old capital, plus the payments received for goods, less the labour earnings paid out, less the actual returns paid out.

Finally, the consumers receive their dividends from the companies and revalue the market capitalisation according to the actual returns paid out.

At this point, the cycle starts again.

As in the companies model, the actual returns paid to the owners (shareholders) that is the payout ratios can depend on whether the surplus revenue generated is greater than the expected returns or less than the expected returns.

For example in model 1D the actual returns paid out are always 70% of the revenue generated. However in models 4A to 4C the actual returns paid out are equal to the real returns produced.

As with the commodities model, it is also possible to put a variable lag in to model the time it takes to install capital.

A further important ingredient in this model is the existence of a 'cash balance' for the householders. This is needed in their role as owners of capital and spenders of money. This cash balance can result as an imbalance of spending outgoing against income received as a consequence of these being dynamic models. If the cash balance is positive then this represents spare cash in the bank. The householders have received more in wages and dividends than they have spent in consumption.

If the cash balance is negative, then this represents a debt to the bank, due to the consumers spending more than they earn.

In the notes following, the cash balance is referred to as H to differentiate it from the capital owned which is now labelled Q. The consumers are assumed to be sensible, so they carry out their consumption based on their total wealth W which is the sum of Q and H, so:

$$C = W\Omega \qquad (4.2a) \qquad \text{or:}$$

$$C = (Q+H)\Omega \qquad (4.2b)$$

So, for example, if H is negative because the consumers have net debt, then consumption is reduced below that judged by the size of Q only.

This model was carried out in Excel, those who wish to go through the maths in detail can paste the model into Excel from appendix 14.9.

4.3 Macroeconomic Models - Results

As expected this model can show different sorts of behaviour, some examples are given below:

Model 4A is the base model, with all the numbers designed to be nice and round. This model has payout ratios of 1 for both the upside and downside. It also allows capital to be added instantly, without any lags. It can be seen from figure 4.3.1 that the output is very stable, and so very dull.

Figure 4.3.1 here

Model 4B, shown in figure 4.3.2 has exactly the same parameters as model 4A, the only difference is that the initial values were different.

Figure 4.3.2 here

This shows just how stable this model is, with the model quickly settling down to equilibrium values. Though even in this stable model it is notable that model 4B needs to go through a number of fluctuations before it arrives at stability (cf figure 1.2.1.4).

But there is a more important difference to note between model 4A and 4B. The parameters of the model are exactly the same, but the equilibrium points are very different. Model 4A started with real capital of 100 units, and settled to an equilibrium at 100 units. Model 4B started with real capital of 400 units, and settled to an equilibrium at about 184 units.

As a consequence, total capital employed at equilibrium in model 4B is much higher than that in model 4A, and more importantly, total employment is higher in model 4B than model 4A. Also the ratio of returns to labour to returns to capital is significantly higher in model 4A.

This is Keynes writ large. Unlike static equilibria, dynamic equilibria can have multiple points of stability. The point of equilibrium that is reached depends on the parameters of the model, but also on the initial conditions. Different initial conditions can give different equilibria even with the same parameters. Once it has reached its equilibrium, the model can stay at that point indefinitely. To change the equilibrium an exogenous force is needed. The model will not rebalance itself to a particular point; a point such as full employment for example. Mass unemployment can continue indefinitely without positive external action.

Model 4C is the most interesting, and most realistic, model.

In this model a time lag has been introduced between capital being purchased and being brought into use. This is identical to the way capital is installed in the commodities models in section 3. Note that the payout ratios are still at unity.

Figure 4.3.3 shows the long term behaviour of the model.

Figure 4.3.3 here

As can be seen the model shows regular cycles of capital being created and destroyed. Again it is important to note that this is a chaotic model, not a stochastic one. There is no stochasticity in this model. All fluctuations in the model are created endogenously from the Lotka-Volterra like differential equations in the model.

Figure 4.3.4 shows the detail of a couple of cycles.

Figure 4.3.4 here

These are real live Minskian / Austrian business cycles. But with one big exception.

It can be seen that real capital K builds up in advance of the total wealth (in this simple model paper wealth; capitalisation is constant), this build up of capital is unsustainable, and so leads to a fall in real capital. Interestingly, although debt (negative cash wealth) is present, this is a lagging variable. In this model debt creation is fuelled by capital growth, not the other way round. The chaotic, bubbly behaviour is not caused by excess credit, it caused by the basic pricing system of capitalism.

Model 4D, shown in figure 4.3.5 below has no lag in the installation of capital. Instead this model has payout ratios of 0.7 on both the upside and the downside.

Figure 4.3.5 here

It is believed that this is a less realistic model, however it does demonstrate how highly chaotic behaviour can be generated in even a very simple model.

Finally model 4E is shown in figure 4.3.6 below. This has a just a small lag of 1 unit for the installation of capital, and payout ratios of 0.8.

Figure 4.3.6 here

Interestingly, it seems that similar results can be achieved without a lag. If both interest rates and payout factors are reduced, an explosive result is also seen.

As can be seen these minor changes in the model are sufficient to create explosive behaviour. This is a true bubble, similar to that of Japan in the 1980s, or the US in the 1920s or in the last decade. Again the cash wealth (debt) is a lagging indicator. It is possible to create explosive bubbles just from the basic pricing system of capitalism.

There is finally one important thing worth noting about the models. The value of the Bowley ratios, β, for the first four models were as follows:

Figure 4.3.7	β
Model 4A	0.75 (exactly)
Model 4B	0.92
Model 4C	0.78
Model 4D	0.85

The Bowley ratio is the ratio of returns to labour to the total returns. The values for models 4C and 4D are averages; the Bowley ratio varies wildly over the course of a cycle in these models.

The numbers above are close to the 'stylised facts' for the Bowley ratio, and are of considerable importance. This is returned to at length in section 4.5 onwards.

4.4 Macroeconomic Models - Discussion

The following section is strongly edited, for a full discussion see the full paper.

As with the previous models, the results above show that a simple combination of classical economics and a dynamic analysis gives interesting results that mirror real economies.

The most interesting result of this model is that the booms and busts are generated internally via an endogenous spiral of creation of wealth. In the model real capital is installed, which generates more paper wealth, which generates more consumption, so feeding into another cycle of wealth creation. The upswing is finally constrained by rising wages making the capital unproductive.

This then generates a downswing of declining wealth, consumption and wages.

This is the normal cycle of capitalism as described by Minsky and the Austrians. Booms and busts are endogenous. Free markets are not inherently stable.

Again, as with the income and company models it noticeable that there are many things that are standard elements of neo-classical or Keynesian economic theory which are simply not needed to produce this macroeconomic model, these include:

- Economic growth
- Population changes

- Technology changes
- Productivity growth
- Investment
- Saving
- Accelerators
- Multipliers
- Shocks (exogenous or endogenous)
- Stochasticity (in any form)
- Different initial endowments (of capital or wealth)
- Utility functions
- Production functions

It has been noted that marginality has worked it's way into the modelling in the form of the pricing curve for labour, this is a reasonable argument, as labour is a commodity that is truly unchangeable in it's supply. However the results of the model show that the detailed form of the curve are completely irrelevant to the model. The curve simply needs to be concave, to ensure that labour costs eventually choke the growth. Within reason, any concave curve will do this. So the actual detail of the calculations of marginality are irrelevant and do not have any influence on the long-term equilibrium, the cycle frequency or the distributions of wealth and income. This is discussed further in section 4.7 of the full paper.

It is also worth considering the 'efficiency' of the economy in this model. This model again creates chaotic behaviour endogenously. There is no stochastic noise in this model. It is politely suggested by the author that a system that endogenously creates booms and busts, with short term creation of excess capital, and far worse; short term destruction of the very same capital, may not, in fact, be allocating capital in a particularly efficient manner.

The validity of the various assumptions of the models are discussed in detail in the full paper.

It can be seen from the model results that economies can behave very differently according to relatively small changes in input parameters.

This is because a system like this can show different regions of behaviour, a general property of Lotka-Volterra and other similar non-linear differential equation models.

Depending on the settings of the variables in the model, there can be three different cases for the outputs.

Firstly, the outputs can be completely stable, quickly going to constant values, this was seen in models 4A and 4B.

Secondly, the outputs can be locally unstable with values constantly varying, but hunting round within a prescribed range of values, this is similar to the lynx and hares Lotka-Volterra model discussed back in section 1.2. This appears to be the way that most normal economies behave. This effect can be caused by the behaviour of capital, either by deliberate hoarding of capital by company managers, or by the time it takes for capital to be installed. The cyclical rise and fall of capital in business cycles is analogous to the cyclical rise and fall of biomass in a biological Lotka-Volterra system. Just as the hares and lynx respond rationally to the available grass, so business investors and speculators react rationally to the opportunities in the economy.

Finally, the outputs can be explosive, moving quickly off to $\pm$ infinity.

In models 4A and 4B these values were 'fixed' to ensure a stable model, in 4C and 4D the parameters were fixed to give a quasi-stable cyclical model, in model 4E they were changed to get explosive models. In the real world it appears that economies operate largely in zones 4C/D, with occasional excursions into zone 4E.

Model 4E suggests that if both interest rates and payout rates are too low then the company sector is too profitable and capital expands exponentially before finally wrecking the whole economy in a glut of capital, see figure 4.3.6 above.

It seems plausible to argue that this reflects what actually happened in the US during the late nineteen twenties, and Japan in the late eighties. Following each of these bubbles the respective economies failed to return to a self-regulating pattern of booms and busts, but appear to have been moved to new equilibrium with much less productive economic patterns. So the economies moved very quickly from a 4E to a poorly performing 4A/B.

A very important point to emphasise in the models above is the absolute lack of stochasticity. While there is certainly a significant element of stochasticity in real markets, the macroeconomic model above contains no stochasticity. The model is not stochastic, it is merely chaotic. Exogenous drivers are simply not needed for quasi-cyclical, or explosive chaotic behaviour.

As discussed above, Lotka-Volterra models have been used in Marxian analysis by Goodwin and others, though the models can be somewhat complex.

The models presented above seem more efficacious than the Goodwin type Lotka-Volterra models, as they don't need:

- population change
- growth in labour force
- technology change
- productivity growth
- inflation (long-term)
- accelerators

all of which are used as standard in the Goodwin and descendant models.

A central problem in the thinking of Goodwin and the researchers that followed Goodwin is in the idea of growth. It appears to have been assumed that to model short-term cycles of growth and decline it was necessary to include long-term economic growth rates. So these models include growth in the labour force, productivity, money supply, etc. This brings in a lot of irrelevancies into the basic model, and make it very hard to build the basic model.

Even without any of the things listed above, natural cycles can occur that build up too much capital.

That is not to argue against the secondary importance of any of the above factors, especially in long-term economic cycles.

Going back to the evidence of Harvie [Harvie 2000] and Barbosa-Filho & Taylor [Barbosa-Filho Taylor 2006], the cycles for the mainland European countries appear to be long term, on a decadal scale; which would suggest a strong role for technology change and productivity growth (though very little for population change). However the cycles for the US and UK appear to show much faster oscillations; of only two to three years. Intuitively it is difficult to see how technology change could operate significantly on such short timescales, and this is more suggestive of the operation of the normal business cycle modelled above.

Indeed the simple model proposed above may be more appropriate for modelling the regular short period cycles of booms and crashes seen in Victorian times.

The important thing to note is that the basic instability in financial markets is much deeper than that proposed by Goodwin. Goodwin style feedbacks may exaggerate this basic cycle, or add longer super-cycles, however in this regard it appears that the basic insight of Minsky and the Austrians with regard to the essential instability of capitalism was correct.

However, although I believe this basic Minskian/Austrian insight is valuable, it is also notable that to build model 4A to E, and create dramatic business cycles, you don't actually need any of the following:

- governments
- fiat money
- fractional reserve banking
- speculators
- Ponzi finance
- debt deflation

or other common elements of the Austrian school or the work of Fisher and Minsky.

Debt, in the form of a negative cash balance, certainly does appear in the cyclical and explosive models. But models 4C and 4D show that the debt follows the cyclical instability of capital rather than the other way round.

I would not wish to understate the importance of debt in exacerbating business cycles, indeed the role of debt appears to be very interesting and important, and is discussed further in 4.6 below. However debt itself is not the prime cause of the business cycles.

Again, it is not suggested that any of the factors listed above are unimportant, however it appears that all the other factors are just potential magnifiers of an underlying inherent instability

The instability is very basic, and, in the short term at least, perfectly rational. The instability arises, as Minsky noted, from the fundamental fact that paper prices of assets are based on projected future cash flows, not on costs of production. This is Minsky's crucial insight, of much greater importance than his analysis of the debt cycle.

This is the same assumption originally proposed in the companies model in section 2.2 above.

This instability naturally produces a growing cycle of apparent wealth, which is turned into excess capital as predicted by Hayek [Hayek 1931] in Austrian business cycles. But contrary to the Austrians, and in line with research data [Kydland & Prescott 1990], the liquidity or excess paper wealth is initially generated within the valuation system of capitalism, not by lax government policy.

Creation of liquidity and monetary growth are endogenous to the basic pricing mechanisms of the finance system. Endogenous creation of financial wealth then feeds back into the creation of more real capital, so creating more financial wealth.

This endogenous creation of financial wealth then gives apparently secure paper assets against which debt can be secured, and of course this debt allows yet more capital creation.

Clearly, if the underlying system is unstable, with endogenous liquidity production à la Minsky; then other factors such as excessive debt, speculation, fractional reserve banking and inappropriate central bank intervention policies will all magnify the size and damage of the underlying cycles. But it is not excessive debt, speculation, fractional reserve banking or poor central bank policy that causes the boom and bust cycles. The cycles are caused by the basic pricing system of capitalism.

Governments may of course fail to calm the markets by extracting liquidity in a timely manner, but it is scarcely the fault of governments that most investors are momentum chasers rather than fundamental analysts.

Just as central banks are expected to control changes in the money supply caused by fractional reserve banking, it seems appropriate that they also need to control liquidity growth caused by Minskian asset pricing. This is discussed in more depth in section 8.2.1 on liquidity in the full paper.

Essentially the company, commodity and macroeconomic models are all simple composites of ideas from Minsky and the Austrian school. The models have Minsky's basic split between 'normal' assets such as goods and services that are priced on a mark-up basis, and financial assets which are priced on the basis of expected future cash flow. Following Minsky, and ultimately Keynes, the expectations of future flows are simplistic projections of present flows [Keen 1995].

Unlike Minsky the models use simple known behaviour of capital to explain the source of instability. In the companies model this was company managers hoarding incoming spare cash, and using it to build more capital. In the commodity model the instability was caused by the time actually taken to build and install new capital. In the macroeconomic models, either or both of these factors could cause instability. In this sense the models follow Austrian ideas. This has the advantage over the Minsky models that you don't need a complex financial system; speculators, Ponzi finance, etc, to form the instability. You can get the instability in pretty much any system where financial assets can be overvalued; this can be Industrial Victorian Britain with its savage business cycles, or even the Roman Empire (see 4.10 in the full paper).

The critical insight of Minsky, in contrast to the Austrians, and seen in these models is that liquidity and new credit are generated endogenously in even the most basic of financial systems. You don't need governments to create excess credit, though certainly they can make things worse. In fact, faced with endogenous credit creation, you do need governments to actively remove credit and liquidity when financial assets become overpriced.

In defining this macroeconomic model, a number of assumptions were made. These are discussed in the full paper.

4.5 A Present for Philip Mirowski? – A Bowley-Polonius Macroeconomic Model

"I mean the stability of the proportion of national dividend accruing to labour, irrespective apparently of the level of output as a whole and of the phase of the trade cycle. This is one of the most surprising, yet best-established, facts in the whole range of economic statistics………Indeed…the result remains a bit of a miracle." [Keynes 1939]

"…no hypothesis as regards the forces determining distributive shares could be intellectually satisfying unless it succeeds in accounting for the relative stability of these shares in the advanced capitalist economies over the last 100 years or so, despite the phenomenal changes in the techniques of production, in the accumulation of capital relative to labour and in real income per head." [Kaldor 1956]

"FUTURE ISSUES - Theory

1. Is there a deep explanation for the coefficient of 1/3 capital share in the aggregate capital stock? This constancy is one of the most remarkable regularities in economics. A fully satisfactory explanation should *not only generate the constant capital share, but some reason why the exponent should be 1/3 (see Jones 2005 for an interesting paper that generates a Cobb-Douglas production function, but does not predict the 1/3 exponent). With such an answer, we might understand more deeply what causes technological progress and the foundations of economic growth."* [Gabaix 2009]

Economists were poorly placed to lecture others on the scientific method; although they trafficked in mathematical models, statistics, and even "experimentation," their practices and standards barely resembled those found in physics or biology or astronomy. Fundamental constants or structural invariants were notable by their absence. [Mirowski 2010]

For most mature economies the ratio of returns to labour to total returns is a near constant that varies between about two-thirds and three-quarters. Young gives a good discussion of the national income shares in the US, while Gollin gives a very thorough survey of income shares in more than forty countries [Young 2010, Gollin 2002].

The constancy of the Bowley ratio is unexpected; in the long run it seems logical that mechanisation and the increasing use of capital would result in the Bowley ratio slowly moving towards zero.

In fact if you analyse the data on a sectoral basis, this is exactly what is happening. Young shows clearly that for agriculture and manufacturing, returns to labour have declined significantly while returns to capital have increased. In the US, returns to labour in agriculture have dropped from nearly 0.8 of total income in 1958 to less than 0.6 by 1996. In manufacturing, the change has been from 0.75 to two-thirds.

This has happened because labour has been slowly displaced by machines in these industries. The fascinating thing is that despite the changes in the Bowley ratios for these two (very large) sectors, the national value of the Bowley ratio for the US as a whole has stayed near constant between 0.69 and 0.66 using the same measures.

The reason for this is that the labour intensive service sector has grown dramatically in size through the same period, and this has kept the national balance of returns to labour and capital very nearly constant.

To derive the Bowley ratio, assume a simple isolated economy at equilibrium, with zero growth, without a state sector, and no debt, then all values of flows and stocks are constant.

At this equilibrium point, the total capital is constant, total income must equal total consumption, then all the definitions below hold. Here e is the earnings paid to labour, π is the profit and can refer to any income from paper assets such as dividends, rent, coupons on bonds, interest, etc, and W is the total capital or wealth represented by the paper assets.

$$\text{Consumption} = \text{Income}$$

$$C = Y$$

$$C = e + \pi \qquad (4.5b) \qquad \text{and:}$$

$$\text{Consumption rate} \quad \Omega = \frac{C}{W} \qquad (4.5c)$$

$$\text{Profit rate} \quad r = \frac{\pi}{W} \qquad (1.3r)$$

$$\text{Income rate} \quad \Gamma = \frac{Y}{W} \qquad (1.3s)$$

$$\text{Bowley ratio} \quad \beta = \frac{e}{Y} \qquad (1.3t)$$

$$\text{Profit ratio} \quad \rho = \frac{\pi}{Y} \qquad (1.3u)$$

$$\beta + \rho = 1 \qquad (1.3v)$$

$$\text{Profit ratio} \quad \rho = \frac{r}{\Gamma} \qquad (1.3w)$$

If we multiply equation (1.3r) by equation (4.5b), then we get:

$$\frac{\pi}{W}C = rY \qquad (4.5e)$$

Substituting from (4.5c) into the left hand side gives:

$$\pi\Omega = rY \qquad (4.5f)$$

Rearranging gives:

$$\frac{\pi}{Y} = \frac{r}{\Omega} \qquad (4.5g)$$

substituting from (1.3u) gives the profit ratio:

$$\rho = \frac{r}{\Omega} \qquad (4.5h)$$

Subtracting both sides from unity gives:

$$1 - \rho = 1 - \frac{r}{\Omega} \qquad (4.5j)$$

or, substituting from (1.3v):

$$\begin{aligned} \beta &= \text{Bowley ratio} \\ &= 1 - \frac{r}{\Omega} \qquad (4.5k) \end{aligned}$$

This is the formula for the Bowley ratio that 'emerged' from my modelling.

At this point, more observant readers may have noticed the similarity of equations (4.5h) and (1.3w) give:

$$\rho = \frac{r}{\Omega} = \frac{r}{\Gamma} \qquad (4.5m)$$

which clearly means:

$$\Omega = \Gamma \qquad (4.5n)$$

from the definitions of Ω and Γ it then follows that:

$$\frac{C}{W} = \frac{Y}{W} \qquad (4.5o) \qquad \text{so:}$$

$$C = Y \qquad (4.5p)$$

which of course was the original definition of (4.5b).

This may appear to be a tautology, but it is firstly worth noting that while this identity is an assumed equality in traditional economics, it is a self-balancing outcome of the GLV and L-V models used in this paper. Consumption is not defined as equal to income or vice versa, consumption of individuals rises and falls with wealth, wealth changes with income and consumption, income depends on consumption. In the models in this paper the dependencies go round in circles, hence the Lotka-Volterra outputs, the equality of total income and consumption naturally falls out at the equilibrium of the model.

And for most economists the above will appear to be a trivial and unimportant accounting identity.

But it isn't.

It is all a question of directionality. Of cause and effect.

For most people it is 'obvious' that consumption follows income, ie that people earn then spend, or that:

$$C = Y$$

Actually it is the other way round:

$$Y = C \qquad \text{or more accurately:}$$

$$\Gamma = \Omega$$

It is the consumption rate Ω that defines Γ; the ratio of total income to capital.

Trivially this is the case in my models, where r and Ω are fixed and Γ is allowed to float. But of course this is not sufficient justification.

The problem with the economic literature with regard to the Bowley ratio is that economists have first defined the profit ratio and Bowley ratio as:

$$\rho = \frac{r}{\Gamma}$$

$$\beta = 1 - \frac{r}{\Gamma}$$

They have then spent the last hundred years or so trying to explain the two ratios above by attempting to look at the microeconomic structure of industry that could affect r and Γ. This has almost entirely revolved around the analysis of 'production functions', the supposed microeconomic relations between capital and labour.

There are however major problems with this approach.

Firstly, real analysis of companies suggests that any meaningful production function needs to be based on high fixed costs and increasing returns, and is far away from the Cobb-Douglas or other standard production functions used in neoclassical economics [Keen 2004, Lee 1999].

Secondly, as the data from Young [Young 2010] shows the relative shares accruing to labour and capital can change quite significantly within individual sectors such as agriculture and manufacturing. This shows that production functions are not giving the required output on a sector-by-sector basis. (Casual inspection of company accounts shows that returns to labour and capital can vary dramatically from company to company.)

The third and most important reason is the problems following the logical steps.

Firstly, traditional economics states that production functions define the relationship between r, the rate of return to capital, and Γ, the rate of total income to capital.

Secondly, traditional economics states that total income is equal to total consumption, so, logically, $\Omega = \Gamma$.

Putting these two statements together logically means that production functions, the microeconomic structure of the commercial sector, define the saving rate Ω (leaving aside r for the moment).

This is very difficult to swallow.

Squirrels save. As do beavers. And also some woodpeckers and magpies. Almost all agricultural societies store grains and other foods to tide them from one harvest to the next. And whether you live in the tropics with alternating wet and dry seasons, or a temperate climate with warm and cold seasons, saving is a biological necessity genetically selected in human beings for its beneficial outcomes.

Saving is a deeply ingrained human behaviour that borders on the compulsive.

Leaving biology aside, traditional economics has well-established logical theories for saving. Lifetime cycles make it logical for young, and especially middle-aged people to save to ensure support in their old age. Whether you look at biology or economics, savings rates are largely exogenous to the economic system.

It stretches credulity to breaking point, to believe that saving and consumption behaviour is ultimately defined by the microeconomic production functions of commercial companies.

The causality works the other way, the systems of capitalism are set up in such a manner that the consumption rate Ω defines Γ, the rate of total income to capital.

When viewed in this way the data of Young makes sense [Young 2010].

In the period Young analysed, consumption rates stayed approximately constant, as did rates of return.

During the same period, both agriculture and manufacturing increased their returns to capital and reduced returns to labour.

Given fixed Ω, to keep things balanced, the economy as a whole was obliged to create new, labour-intensive, industries to ensure that returns to labour were maintained as a whole.

All those cappuccino bars and hairdressers were created by the economy; by entropy, to ensure that the Bowley ratio remained equal to $1-(r/\Omega)$.

Given r and Ω as positive ratios, and that Ω is normally larger than r, then the Bowley ratio will normally be a fraction between 0.5 and 1.0.

It is straightforward to check equation (4.5k) against reality. A suitable long-term profit rate could be anywhere between long-term interest rates and long-term real stock-market returns. Long-term real interest rates are generally in the region of 2% to 5% [Homer & Sylla 1996, Measuring Worth]. Long-term stock-market returns appear to be in the region of 7% to 8% [Campbell 2003, Ward 2008].

Consumption is typically about 60% of gdp [Miles & Scott 2002, section 2.2, fig 2.3]. While non-residential capital stock is typically 2.5 to 3 times gdp [Miles & Scott 2002, section 5.1 & 14.1]. Taken together this would give Ω, the consumption rate as a proportion of capital a range of about 0.2 to 0.25.

Substituting into equation (4.5k) this then gives a possible range of values for the Bowley ratio of between 0.60 and 0.92

Clearly this range is a little on the high side when compared with the 'stylised facts' of observed Bowley Ratios in the real world varying between the values of 0.5-0.75.

We are however in the right ballpark.

I find it difficult to believe that I am the first researcher to propose that the Bowley ratio should be defined by:

$$\beta = 1 - \frac{r}{\Omega} \qquad \text{rather than:}$$

$$\beta = 1 - \frac{r}{\Gamma}$$

However, I have not been able to find any other proposal of this relationship, and the recent writings of Gabaix, Young and others suggest that this is the case. If I am the first to do so I am happy to take the credit. If not I would be happy to update this paper appropriately.

4.6 Unconstrained Bowley Macroeconomic Models

The following section is speculative, the role of the cash balance H is not well defined in the macroeconomic models above.

When the condition of zero cash balance, is removed a more complex formula for the Bowley ratio emerges from the model:

$$\beta = \frac{\Omega + \Omega(H/Q) - r}{\Omega + \Omega(H/Q)}$$

$$= \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)} \qquad (4.6a)$$

where H is the cash balance (wealth held in the form of cash or negative debt) and Q is the wealth held as capital.

As in the previous section it is fairly trivial to derive equation (4.6a) from first principles.

As before, when the model is at equilibrium, all values of flows, stocks and debts are constant. At this point, if the values of capital Q and cash H are to be constant, then the total income must equal the total outgoings, so, as before:

$$C = Y$$
$$= e + \pi \qquad (4.6b)$$

However this time, in the original model, in equation (4.2b), we defined the consumption ratio Ω as:

$$\Omega = \frac{C}{(Q + H)} \qquad \text{so,}$$

$$\Omega(Q + H) = C \qquad \text{or, substituting from (4.6b):}$$

$$\Omega(Q + H) = Y \qquad (4.6d)$$

again, the profit rate is defined by:

$$\pi = rQ \qquad (4.6e)$$

If we multiply equation (4.6d) by equation (4.6e), then we get:

$$\pi\Omega(Q + H) = rQY \qquad (4.6f)$$

Rearranging gives:

$$\frac{\pi}{Y} = \frac{rQ}{\Omega(Q + H)} \qquad \text{or:}$$

$$\rho = \frac{rQ}{\Omega(Q + H)} \qquad (4.6g)$$

Subtracting both sides from unity gives:

$$1 - \rho = 1 - \frac{rQ}{\Omega(Q + H)} \qquad (4.6h) \qquad \text{or from (1.3v):}$$

$$\beta = \frac{\Omega(Q + H) - rQ}{\Omega(Q + H)} \qquad \text{or,}$$

$$\beta = \frac{\Omega Q + \Omega H - rQ}{\Omega Q + \Omega H} \qquad \text{dividing by } \Omega \text{ and Q;}$$

$$\beta = \frac{1 + (H/Q) - (r/\Omega)}{1 + (H/Q)} \qquad (4.6a)$$

Once again the base equation here is (4.6g) which is the ratio of returns from capital, to total returns.

It can be seen from equation (4.6a) that the Bowley ratio can be manipulated by changing the value of the cash balance H.

If the cash balance is positive and increasing, Bowley's ratio just heads closer and closer to unity, good for workers, bad for capitalists.

More interestingly, if H is negative, a debt, and the size of the debt is increased, then the size of both the numerator and denominator reduce, however the value of the numerator reduces more rapidly than the size of the denominator, and the Bowley ratio slowly decreases. At least at first.

If debt is allowed to continue increasing, then a rather dull function suddenly becomes more interesting. Firstly the Bowley ratio drops rapidly to zero, and then shortly afterwards heads off to negative infinity.

In the model itself it isn't possible to reach these points; as the Bowley ratio heads to zero the model becomes unstable, and explosive – the economy blows up in an entertaining bubble of excess real capital and even more excess debt. This may sound familiar.

There is a further consequence of this model. In this model the role of debt gives a direct output to the Bowley ratio. As was found in section 1.6 of the full paper, the Bowley ratio in turn gives a direct output to the parameters of the GLV income distribution.

So, if the above models hold, there is a direct link from levels of debt in the economy to the levels of inequality. Specifically, increased levels of debt lead to increased levels of inequality.

Intuitively this seems plausible. Looking back over the last century, especially at the US, the first part of the century was associated with high levels of inequality, and high levels of leverage, which ultimately resulted in the Wall Street crash and the depression. In reaction to this, from the 40's to the 70's, leverage was strictly controlled, and also income distribution was much more equitable. From the 70's to the end of the 20th century, increased financial deregulation, and increased leverage, went hand in hand with increased inequality. It should be straightforward to check this against historical data.

If economic data does support equation (4.6g), or a variant of it, then this raises interesting discussions on the role of debt in a national economy. The history of the last forty years has been one in which neoclassical economists have argued forcefully for the liberalisation of financial markets under the assumption that deregulation would allow deeper and cheaper financial markets and that self-regulation would ensure a natural balancing of an equilibrium. Equation (4.6g) begs to differ.

Equation (4.6g) dictates that persuading governments to allow greater leverage merely allows benefits to the owners of capital, while simultaneously moving towards a more unstable equilibrium that coincidentally increases overall wealth inequalities.

Equation (4.6g) suggests that control of the national level of leverage can provide three separate economic benefits. Firstly for the working of the economy there will be a optimum level of debt that allows liquidity and provides capital for genuine economically productive investment. Secondly, by preventing extreme levels of debt financial instability can be prevented. Thirdly, the level of debt may be reduced to achieve reduced levels of inequality.

If the third item above is tackled successfully then the second becomes irrelevant, so the debate regarding the appropriate level of debt becomes a trade off between the first and third items.

While the income distribution requirements suggest an elimination of debt, this is clearly not practical for a well functioning economic system. While much investment is funded directly from cashflow, if the economy is to grow successfully non-financial firms clearly need access to debt financing for major capital investments.

Although the work of Minsky is not quantitative in nature, his characterisation of the phases of debt build up is clear and easy to relate to real economic cycles. If equation (4.6g) above is found to be applicable, it should be possible to look through past economic cycles and note where debt moved from a useful point; of providing funds for investment and price finding speculation, to turning into a self-sustaining provider of bubble finance. This would then provide central banks with a guide to controlling financial markets for the benefit of the economy as a whole.

In section 4.7 of the full paper there is a discussion of the conceptual importance of equations (4.5h) and (4.6g). In section 4.8 of the full paper there is a detailed discussion of why the Bowley ratio prevents human beings being freed from long working hours; why the futurologists predictions of a life of leisure have been frustrated.

Sections 4.9 of the full paper discusses an extension of the macroeconomic model to include separate financial and non-financial sectors. Sections 4.10 of the full paper discusses two macroeconomic models in parallel are discussed as an international finance model. This gives a model that explains the behaviour seen by Reinhart and Rogoff in 'This Time is Different' [Reinhart & Rogoff 2009] and described by Michael Pettis in the 'Volatility Machine' [Pettis 2001]. Section 4.11 of the full paper discusses the introduction of governments and monetary policy in the macroeconomic model.

11. The Logic of Science

In their abstract to 'Worrying trends in econophysics' Mauro Gallegati, Steve Keen, Thomas Lux and Paul Ormerod wrote:

'Our concerns are fourfold. First, a lack of awareness of work that has been done within economics itself. Second, resistance to more rigorous and robust statistical methodology. Third, the belief that universal empirical regularities can be found in many areas of economic activity. Fourth, the theoretical models which are being used to explain empirical phenomena. The latter point is of particular concern. Essentially, the models are based upon models of statistical physics in which energy is conserved in exchange processes. There are examples in economics where the principle of conservation may be a reasonable approximation to reality, such as primitive hunter–gatherer societies. But in the industrialised capitalist economies, income is most definitely not conserved. The process of production and not exchange is responsible for this. Models which focus purely on exchange and not on production cannot

by definition offer a realistic description of the generation of income in the capitalist, industrialised economies.' [Gallegati et al 2006]

I am slightly embarrassed to admit that, due to both time constraints and limited experience in econometrics, the present paper remains significantly remiss with regard to the second criticism above.

But, then again, to rephrase Ernest Rutherford; if you need to use statistics to prove your theory, you ought to have thought of a better theory.

In the event of some party choosing to award me remuneration for my ongoing research I would hope to remedy these shortcomings in future papers.

However I believe the present paper has come a long way in answering the other criticisms.

In particular, I believe criticisms one and four have been fully addressed in this paper.

I believe however that the authors' third criticism is fundamentally flawed.

It is the nature of science that a field can appear complex and difficult to make any sense of until a significant insight can bring sudden clarity. It has taken time for physicists to bring this clarity to economics, but to physicists, the multi-body nature of economic and financial systems meant that the belief that universal empirical regularities would be explained was only a matter of time. It is this insight that drove Champernowne half a century ago. It is this insight that resulted in Wright, Souma & Nirei and myself independently producing similar models near simultaneously.

It is a canard among economists that physicists have moved into economics and finance due to the lack of job opportunities in mainstream physics. This may be the case for quants in the City, but it is not the case for econophysics researchers. For the research oriented physicist the attraction is a mathematical field that has not been effectively analysed, but that clearly has parallels with other fields that have been regularised. Finding wide open research areas like this in the mainstream sciences or engineering is difficult. Economics offers the low-hanging fruit of major new research findings, that is if you can truly describe a field full of watermelons as 'low-hanging'.

Indeed the '*universal empirical regularities'* pooh-poohed by Gallegati et al where always there.

Wealth distributions, company size distributions, and the split of the returns from labour and capital are all long-standing 'anomalies' within economics. Economists such as Schumpeter and Gabaix have noted these regularities [Gabaix 2009]. Why almost all other economists, even heterodox economists such as Gallegati et al, have shown such disinterest in investigating these recurrent and profound features of economics has always been puzzling to physicists.

Economics has systematically treated such persistent 'anomalies' as anomalies, ignoring raw data while retreating into the comforts of intellectual hypothesis, whether this be neoclassical, Keynesian, Marxian, behavioural or other. Even in areas such as post-Keynesian or behavioural economics, were data collection has become something of a fetish, the flavour of the data collection still gives the feel of data being ransacked to prove the previously held opinions of the researcher, rather than the data being looked at and analysed as it is found.

It is precisely by investigating 'anomalous' but persistent data outputs that the natural sciences have progressed.

By definition, if data output is persistent, it is not 'anomalous'.

If data output is persistent, it is normal.

It may be 'anomalous' with regard to current theory. But that simply makes the current theory by definition 'anomalous', not the data.

In these circumstances the theory must be abandoned, not the data.

Einstein, for example, is usually characterised as a theoretical physicist. But his biggest single insight (amongst many) was to treat the experimental fact of the constancy of the speed of light as a given.

From this he abandoned 'common sense' and simply worked out the mathematical consequences of this fact. Thus was relativity born.

Economists seem to prefer the route of Einstein's peers, forever producing more complex theories to substantiate the existence of a hypothetical aether.

Science can not be built simply on common sense, intuition and intellectual rigour.

Science must start with the observed facts if it is to make progress.

This, at a much deeper level than that intended by Jaynes, is the logic of science.

For any multi-body system, entropy has to be the guiding force, it has taken time for physicists and mathematicians to get to the root of this, mainly because the entropy was dynamic path entropy rather than static state entropy, but the driving power of entropy in economics is immediately obvious to anybody who has a passing understanding of entropy.

Economics is a specialised study of entropy. It is a branch of thermodynamics, a branch of physics.

Like information theory, in fact even more so than information theory; economics is a very complex, interesting and important subject in its own right. But nonetheless it is a subset of thermodynamics.

It is the application of dynamics and statistical mechanics to political economy.

It is econodynamics.

11.1 Afterword

As noted in the introduction, this paper is a condensed extract of the full paper 'Why Money Trickles Up'. The full paper applies the same basic model to explain the power tail seen in wealth and income distributions. The full paper also contains extensive background material on chaos, statistical mechanics, entropy and heterodox economics and finance. The abstract and paper structure for the full paper are given below in section 11.2.

The full paper can be downloaded at econodynamics.org.

It was noted in the introduction that this paper was researched and written in a little over a year, without financial support or academic supervision.

Foolishly, I have gone against a basic conclusion of this paper, and spent a significant portion of my own capital in producing it.

If you have found the paper of interest or value, any donation to defray the costs of writing it, no matter how small, would be gratefully received.

Those who wish to make a donation can do so by clicking on the Paypal link below:

click here to make donation

(Paypal accept all major credit cards, you do not need to have a Paypal account.)

11.2 Abstract & Structure of full paper 'Why Money Trickles Up'

Abstract

This paper combines ideas from classical economics and modern finance with Lotka-Volterra models, and also the general Lotka-Volterra models of Levy & Solomon to provide straightforward explanations of a number of economic phenomena.

Using a simple and realistic economic formulation, the distributions of both wealth and income are fully explained. Both the power tail and the log-normal like body are fully captured. It is of note that the full distribution, including the power law tail, is created via the use of absolutely identical agents.

It is further demonstrated that a simple scheme of compulsory saving could eliminate poverty at little cost to the taxpayer. Such a scheme is discussed in detail and shown to be practical.

Using similar simple techniques, a second model of corporate earnings is constructed that produces a power law distribution of company size by capitalisation.

A third model is produced to model the prices of commodities such as copper. Including a delay to capital installation; normal for capital intensive industries, produces the typical cycle of short-term spikes and collapses seen in commodity prices.

The fourth model combines ideas from the first three models to produce a simple Lotka-Volterra macroeconomic model. This basic model generates endogenous boom and bust business cycles of the sort described by Minsky and Austrian economists.

From this model an exact formula for the Bowley ratio; the ratio of returns to labour to total returns, is derived. This formula is also derived trivially algebraically.

This derivation is extended to a model including debt, and it suggests that excessive debt can be economically dangerous and also directly increases income inequality.

Other models are proposed with financial and non-financial sectors and also two economies trading with each other. There is a brief discussion of the role of the state and monetary systems in such economies.

The second part of the paper discusses the various background theoretical ideas on which the models are built.

This includes a discussion of the mathematics of chaotic systems, statistical mechanical systems, and systems in a dynamic equilibrium of maximum entropy production.

There is discussion of the concept of intrinsic value, and why it holds despite the apparent substantial changes of prices in real life economies. In particular there are discussions of the roles of liquidity and parallels in the fields of market-microstructure and post-Keynesian pricing theory.

Structure of the Paper

Part A of this paper discusses a number of economic models in detail, Part A.I discusses a number of straightforward models giving results that easily accord with the real world and also with the models of Ian Wright. Part A.II discusses models that are more speculative.

Part B discusses the background mathematics, physics and economics underlying the models in Part A. The mathematics and physics is discussed in Part B.I, the economics in part B.II, the conclusions are in part B.III. Finally, Part C gives appendices.

Within Part A; section 1 discusses income and wealth distributions; section 1.1 gives a brief review of empirical information known about wealth and income distributions while section 1.2 gives background information on the Lotka-Volterra and General Lotka-Volterra models. Sections 1.3 to 1.5 gives details of the models, their outputs and a discussion of these outputs.

Section 1.6 discusses the effects that changing the ratio of waged income to earnings from capital has on wealth and income distributions.

Sections 1.7 and 1.8 discuss effective, low-cost options for modifying wealth and income distributions and so eliminating poverty.

Finally, section 1.9 looks at some unexplained but potentially important issues within wealth and income distribution.

Sections 2.1 to 2.4 go through the background, creation and discussion of a model that creates power law distributions in company sizes.

Sections 3.1 to 3.4 use ideas from section 2, and also the consequences of the delays inherent in installing physical capital, to generate the cyclical spiking behaviour typical of commodity prices.

Sections 4.1 to 4.4 combine the ideas from sections 1, 2 and 3 to provide a basic macroeconomic model of a full, isolated economy. It is demonstrated that even a very basic model can endogenously generate cyclical boom and bust business cycles of the sort described by Minsky and Austrian economists.

In section 4.5 it is demonstrated that an exact formulation for the Bowley ratio; the ratio of returns to labour to total returns, can easily be derived from the basic macroeconomic model above, or indeed from first principles in a few lines of basic algebra.

In section 4.6 and 4.7 the above modelling is extended into an economy with debt. From this a more complex, though still simple, formulation for the Bowley ratio is derived. This formulation suggests that excessive debt can be economically dangerous and also directly increases income inequality. The more general consequences of the Bowley ratio for society are discussed in more depth in section 4.8.

In section 4.9 two macroeconomic models are arranged in tandem to discuss an isolated economy with a financial sector in addition to an ordinary non-financial sector. In section 4.10 two macroeconomic models are discussed in parallel as a model of two national economies trading with each other.

To conclude Part A, section 4.11 introduces the role of the state and monetary economics, while section 4.12 briefly reviews the salient outcomes of the modelling for social equity.

In Part B, section 6.1 discusses the differences between static and dynamic systems, while section 6.2 looks at the chaotic mathematics of differential equation systems. Examples of how this knowledge could be applied to housing markets is discussed in section 6.3, while applications to share markets are discussed in section 6.4. A general overview of the control of chaotic systems is given in section 6.5.

Section 7.1 discusses the theory; 'statistical mechanics', which is necessary for applying to situations with many independent bodies; while section 7.2 discusses how this leads to the concept of entropy.

Section 7.3 discusses how systems normally considered to be out of equilibrium can in fact be considered to be in a dynamic equilibrium that is characterised as being in a state of maximum entropy production. Section 7.4 discusses possible ways that the statistical mechanics of maximum entropy production systems might be tackled.

Moving back to economics; in section 8.1 it is discussed how an intrinsic measure of value can be related to the entropy discussed in section 7 via the concept of 'humanly useful negentropy'.

Section 8.2 discusses the many serious criticisms of a concept of intrinsic value in general, with a discussion of the role of liquidity in particular.

Section 9.1 looks at theories of supply and pricing, the non-existence of diminishing returns in production, and the similarities between the market-microstructure analysis and post-Keynesian pricing theory. Section 9.3 looks for, and fails to find, sources of scarcity, while section 9.4 discusses the characteristics of demand.

In section 10 both the theory and modelling is reviewed and arranged together as a coherent whole, this is followed by brief conclusions in section 11.

Sections 12 to 16 are appendices in Part C.

Section 12 gives a history of the gestation of this paper and an opportunity to thank those that have assisted in its formation.

Section 13 gives a reading list for those interested in learning more about the background maths and economics in the paper.

Section 14 gives details of the Matlab and Excel programmes used to generate the models in Part A of the paper.

Sections 15 and 16 give the references and figures respectively.

Part C - Appendices

12. History and Acknowledgements

Between 1980 and 1982 I was taught A-level physics by Malcolm Ruckledge using the innovative Nuffield Foundation Physics course. This was a powerful combination of an outstanding teacher with outstanding material. The section on statistical mechanics was particularly well written and taught, and gave me an early and profound intuitive insight into the power and simplicity of entropy. I suspect this paper would not have been written without this insight.

Sometime in my first year studying physics at the University of Manchester, in 1992/3, while looking at a picture of the Maxwell-Boltzmann distribution of molecular velocities on a blackboard, it occurred to me that wealth in a society was shared out in a similar manner; a lot of people with a little wealth and a few with a lot of wealth. It further occurred to me that the underlying systems, involving a lot of freely interacting particles/individuals, where fundamentally similar. At the time I imagined this was a unique and very clever insight, however it turned out that a lot of other physicists and mathematicians have had similar insights, some preceding mine by many decades.

After this, nothing very much happened for a decade or so, though the idea refused to go away, and being by nature an engineer at heart, I thought a lot about how income and wealth inequality might be tackled as well as to why it exists.

In 2003 I had a letter published in the New Scientist. This encouraged me to take my ideas more seriously, and while working abroad in 1995 I had the opportunity to write down my ideas at that stage into a fairly amateurish paper.

On returning to the UK I circulated the paper around various individuals I thought might be interested. The paper was greeted on a spectrum that largely went from disinterest through to derision.

One exception was Michael Stutzer, who suggested I forward it to Duncan Foley, with whom I had a brief but very rewarding correspondence. I remain very thankful to both these individuals and especially to Duncan Foley for encouraging my work even when it was at this very early and amateurish stage.

After this nothing very much happened again for some years, as I lacked the skills, in both economics and mathematics to take the work forward. I did however read a paper by Ayres & Nair 'Thermodynamics and economics' which I found very useful in linking the concept of entropy to the economic concept of value.

This changed in August 2000 when, via the New Scientist, I discovered the work of Bouchaud & Mézard and other researchers, primarily physicists but also some heterodox economists, working in the new field of econophysics. The majority of the work was in the field of asset pricing in finance, but there was also a parallel stream looking at wealth and income distributions.

Over the next few years I attended a number of econophysics and related conferences where I learned a lot more about both the maths and economics from the other participants.

During this period I was given support and guidance, from Steve Keen, Thomas Lux and others, but most particularly from Juergen Mimkes, for which I would like to give thanks. Thomas Lux gave me some very

useful insight into the real meaning of value and wealth that helped to generate the ideas in this paper. Steve Keen gave interesting discussions on economics and also pointed me in the direction of James Galbraith who was also very supportive.

As stated in the introduction I met Wataru Souma at the Econophysics of Wealth Distributions conference at Kolkata in 2005. I almost certainly attended his lecture on his paper 'Universal Structure Of The Personal Income Distribution'. I found Souma & Nirei's model complex and difficult to follow, and did not knowingly use it further.

Judging from the pile of papers that I rediscovered it in; it appears that I read Ian Wright's 'The Social Architecture of Capitalism' some time shortly after the Kolkata conference. I remember reading this paper quite clearly, as the style of the paper was unusual. The paper is very strongly a modelling paper, with very little formal mathematical content. This resulted in my finding it very difficult to make much sense of, and in fact I didn't understand the paper until some years later. I also, at the time, found the Marxian approach very naïve and off-putting, particularly in the insistence on the use of the labour theory of value. This seemed to me plainly wrong; so at this stage I dumped this paper in the 'irrelevant' pile and forgot about it. That was a big mistake.

In 2006 it was suggested to me that the general Lotka-Volterra distribution might make a good fit to some high quality income data I had acquired from the UK Statistical Office. It turned out that the data did fit the GLV exceptionally well; better than alternative distributions.

As a scientist, this dictated that building models along the lines of the GLV would be the most sensible way forward.

By this stage, my knowledge of economics had expanded a little, and I was somewhat dismayed by the naivety and complexity of the approaches taken to economics by most physicists. It seemed to me that power law distributions, and gross inequality, had a universality through geography and more importantly history (cf the paper regarding inequality in ancient Egypt [Abul-Magd 2002]), and that they appeared to be valid in any society where wealth, including land, was traded.

This could be contrasted with, for example, community owned land systems in Africa, which though associated with general poverty appeared to be characterised by low levels of inequality. In my view any model for wealth distributions should be able to accommodate payments to capital in the broadest sense, whether this be via dividends, interest rates, or rent on land and property.

With this in mind I attempted to fit, in the simplest way possible, basic economic concepts to two different generating equations that I was aware were capable of producing GLV distributions. These two systems were the exchange system of Slanina and the GLV system of Levy & Solomon. I wrote a note and circulated it to a number of academics in early 2006, I have reproduced the note in full below in section 12.1.

Unfortunately, none of the academics proved interested in my proposals. Also unfortunately, I did not send the note to Wright or Souma & Nirei, as it had been some time since I read their papers, and I didn't consciously connect them to this present work.

I lacked the mathematical or programming skills to take this forward, so once again, nothing much happened for a few years.

In 2009, in the middle of the post-credit-crunch recession, I took the opportunity to start an MSc in Finance at Aston University. Due to some very unfortunate circumstances I was unable to complete the course.

However in the two terms I attended the course I acquired a lot of useful knowledge regarding basic finance and economics. I would also like to give thanks to Patricia Chelley-Steely for giving me important insights into the role of market-microstructure in general and liquidity in particular.

I was also able to gain invaluable assistance from George Vogiatzis and Maria Chli with regard to producing simulations of my models proposed in 2006. The exchange model proved difficult to construct. However, in March 2010 Maria and George produced the first Matlab model for me based on the GLV model in the second part of my 2006 note. Somewhat to my surprise, this produced a perfect GLV distribution on its first run, though no power law.

It turned out that, to generate the power law, the profit ratio had to be increased substantially from the initial 5% proposed to somewhere near 50%. A little investigation revealed that the returns to capital where indeed on this scale, and so this was realistic.

At this point George and Maria politely, but firmly, suggested that I conquer my technophobia and learn to program in Matlab myself. I followed their advice and discovered that it is a lot easier than other programming languages I had encountered. From the first programme, I produced all the other programmes in this paper in short order, with almost all programming work being done in May 2010. I remain deeply indebted to Maria and George for their initial assistance and support with this work.

The income model followed naturally from the wealth model. The companies model followed naturally from the wealth and income models. The commodity model followed naturally from the companies model.

During the modelling process I was rereading Steve Keen's 'Debunking Economics' and had also read some of the Goodwin modelling work while investigating the ratio of returns to labour and capital. It seemed to me that by combining the wealth, company and commodity models it would be possible to generate a much simpler but effective Goodwin style macroeconomic model. This proved to be the case, with a resultant simple base model that appeared to produce Minskian/Austrian cycles endogenously.

At some point after the modelling was largely complete, while rereading a large volume of papers I had collected over the years, I reread Wright's 'The Social Architecture of Capitalism'. For the second time I found it difficult to follow, and found the labour theory of value difficult to accept. However something in the paper was nagging at me. I reread the paper for a second time, more carefully; and slowly realised that, though coming from a completely different angle, Wright had built a model that was both making the same base assumptions as my own, and producing many of the same outputs. Indeed, in many ways Wright's models produced better results than my own.

Given the very different ways that Wright and myself produced our models, I believe that my approach was not influenced by Wright. My original proposals of 2006 were deliberately, mathematically based on the GLV, and were also focused on a financial sector with returns paid on capital. Wright's models are significantly different to my own, most notably in not involving a financial sector. Also, unlike the present paper Wright takes a 'black box' and 'zero intelligence' approach to modelling which eschews formal fitting of the models to mathematical equations.

Despite this belief, I am obliged to accept that I may have been influenced subconsciously by Wright's work.

Much later in the writing of this paper, close to it's completion, I reread the work of Souma & Nirei. Again I found the complexity of the mathematical approach of Souma & Nirei very difficult to follow, and I believe this complexity is unnecessary, and that my own approach is more useful as a basis for analysing economics. However the parallels between their work and my own are significant. Most notably Souma & Nirei use consumption as a dissipative part of their model in a way that is almost identical to my own models.

They also use capital as a main source of new wealth in their model, which is analogous to my own, though less strongly than with consumption. Souma & Nirei use capital growth as the main form of supplying new wealth to their model. They justify this by using supporting data from the Japanese economy. While this may have seemed sensible at the time, given the collapse of the Japanese stock-market and property prices over the last two decades, this now looks less sensible. Although I believe that capital growth can form a part of wealth generation, on a long-term cyclical basis this is likely to be very small. I believe that my simple model of returns to capital in the form of interest, dividends and rent is a better basis for future economic modelling.

As with Wright, I do not believe I was influenced directly by Souma & Nirei. My first model in 2006 was a simple exchange model, quite different to that of Souma & Nirei, while I generated the second model by simply substituting what to me were the most obvious and simple economic terms into Levy & Solomon's generating equation. Indeed my original model was a little over-complex and significantly different to that of Souma & Nirei.

However, even more so than Wright's work, the parallels between the models of Souma & Nirei and my own are striking. And the possibility that I was subconsciously assisted by their work seems significant.

I would like to state in the strongest terms that I believe that the work of Wright, Souma & Nirei is of considerable importance. These three academics have been able to bridge the gap between the physics and the economics in a way that no other academics have been able to. Also they all carried out this work prior to my own.

Where my own work differs to that of the gentlemen above is that it has a clear mathematical basis, unlike that of Wright, and that the mathematical basis is dramatically simpler than that of Souma & Nirei.

It is my hope that Wright, Souma, Nirei and myself can share the credit for finally bringing an effective mathematical and modelling approach to the understanding of economics.

12.1 Proposed Models 2006

Pair exchange process, after Slanina;

$W_{i,t+1} = W_{i,t} + \beta_{ij} - \beta_{ji} - p_i + r * W_{i,t}$

$W_{j,t+1} = W_{j,t} + \beta_{ji} - \beta_{ij} - p_j + r * W_{j,t}$

β_{ij} would be a good or service received,
β_{ji} would be money exchanged for the good or service,
(or vice versa) you could make this more 'economist friendly' by using:
β_{gs} for a good or service received,
β_m for money exchanged for the good or service,

typically β_{ij} would be a factor smaller than $W_{j,t}$ in size

$\Delta\beta = \beta_{ij} - \beta_{ji}$
is a small random difference in wealth due to the exchange not being exactly equal, typically $\Delta\beta$ would be a few percent of β_{ij} (economists would argue that $\Delta\beta$ would be equal to zero at equilibrium, I believe this is not the case, however it is much easier just to argue that there will be small random differences in the wealth exchange, which is a very plausible assumption) I see the $\Delta\beta$'s as the main stochastic driver in this model.

p_i

is the profit taken by a third party. If I buy a car directly off you, then p_i equals zero, but if I buy a car off you via e-bay, a small percentage of β_{ij}; p_i and/or p_j is taken by e-bay. (In e-bay's case, the seller is charged, so $p_i = 0$). Ignoring the example of e-bay, I would initially model this by assuming that all p_i's are a fixed small percentage of the exchange. So:

$p_i = \beta_{gs} * p_{rate}$

r
is the interest rate (factored down to a weekly or daily rate, whatever Δt is) Annual real interest rates (after inflation) are very stable, varying between 0.5 and 4% (annual) over long time periods. I would also initially model this as a small fixed percentage. (To get a working model with equations that balance it may be necessary to have a fixed relationship between and p_{rate} and r ;
$p_{rate} = const * r$)

I do not see any reason to make the r 's a distribution set. Most peoples investments are stable, poor peoples especially so. Rich people will only hold a portion of investments in riskier, more variable funds. I would only really see a need to introduce a distribution set if it was the only way we could generate the necessary curve.

So in this model the change in wealth comes from a small random element from the exchange, a small element taken in profit, and a small gain of interest which, crucially, is proportional to current wealth.

From a max entropy type approach I would then add the following two conditions:

$\Sigma W_{i,t} = \Sigma W_{i,t+1}$

ie, all wealth is conserved (ie. there is no economic growth or recession).

And:

$\Sigma p_i = \Sigma r * W_{i,t}$

ie, all profit is recycled as interest on peoples wealth.

In this model the stochastic variability comes from the wealth exchanges; the $\Delta\beta$'s. This combined with the assumption of conservation of wealth would provide a boltzmann type distribution if profits p_i and interest r were equal to zero.

I believe the extra terms of profit and interest will be a circular reinforcing mechanism that should produce the power tail.

If you can solve, this or something similar, hopefully you will get a wealth distribution that is a GLV with alpha = 1.5

<u>GLV type process;</u>

$W_{i,t+1} = W_{i,t} + Inc_i * \Delta t - p_{Inc} - Con_i * \Delta t - p_{Con} + r * W_{i,t}$

Inc_i
is waged income; income from employment. Realistically I would expect this to be a stable distribution, very much on the lines of Juergen's arguments. (http://arxiv.org/abs/cond-mat/0204234)

p_{Inc}
is the small profit taken by the employing organisation. Modelled as previous model.

Con_i

is consumption, which includes food, clothes, new cars, petrol, rent[#], mortgage payments[#], holidays, etc. ([#] not completely sure about these two). Consumption is the big variable, and is where I would expect the stochastic element to come in strongly.

p_{Con}
is the small profit taken by the shop, landlord, building society, etc.

r
As previous model.

Again, from a max entropy type approach, I would then add the following two conditions:

$\Sigma W_{i,t} = \Sigma W_{i,t+1}$

again, all wealth is conserved.

And:

$\Sigma (p_{Inc} + p_{Con}) = \Sigma r * W_{i,t}$

Again; all profit is recycled as interest.

From this equation you can derive something like:

Total Income = I_j

$I_j = [Inc_i + (r * W_{i,t} / \Delta t)] = [\text{wages + interest, etc}]$

$I_j = [Con_i + ([(W_{i,t+1} - W_{i,t}) + (p_{Inc} + p_{Con})] / \Delta t)]$

If you can solve this or something similar, hopefully you will get an income distribution that is a GLV with alpha = 4 to 5.

13. Further Reading

See full paper for details.

14. Programmes

See full paper for details.

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16. Figures

Figure 1.3.5

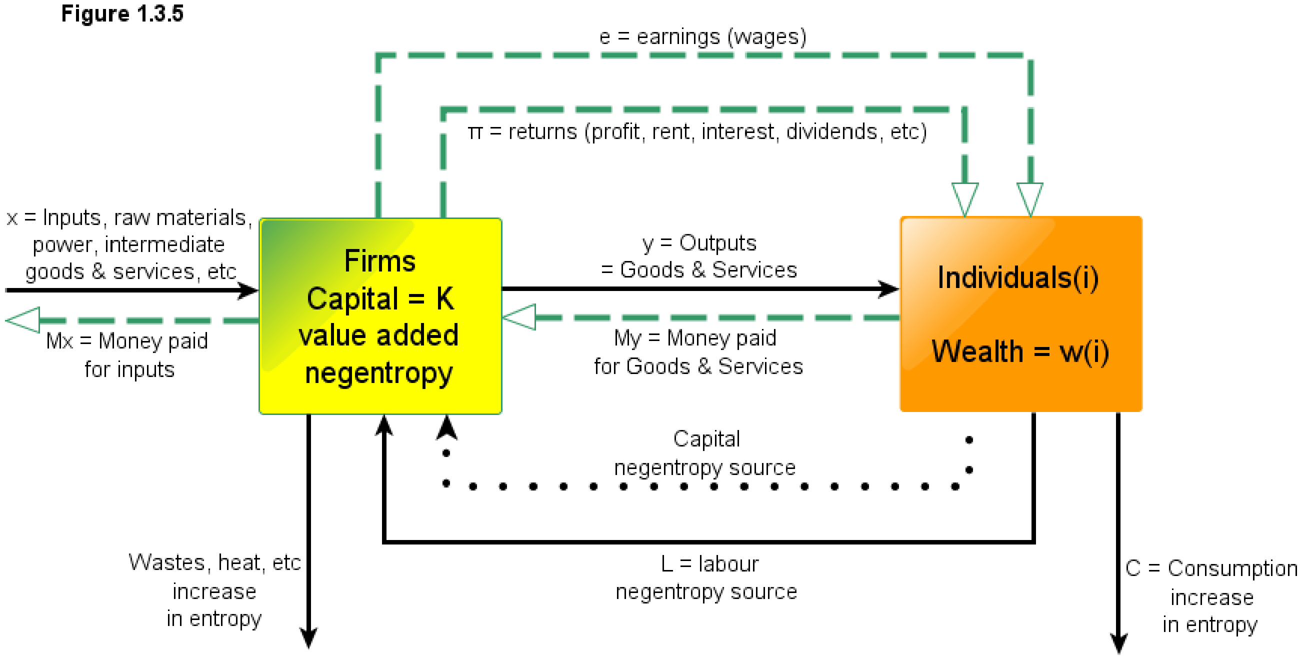
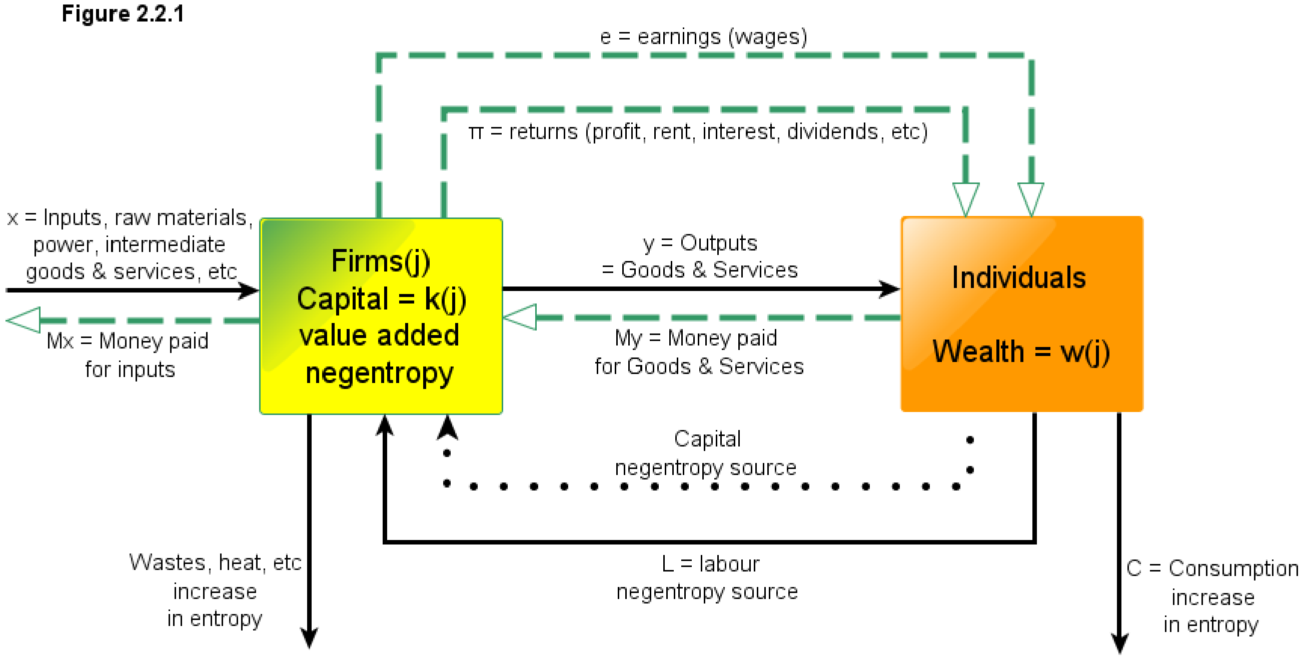


Figure 2.2.1



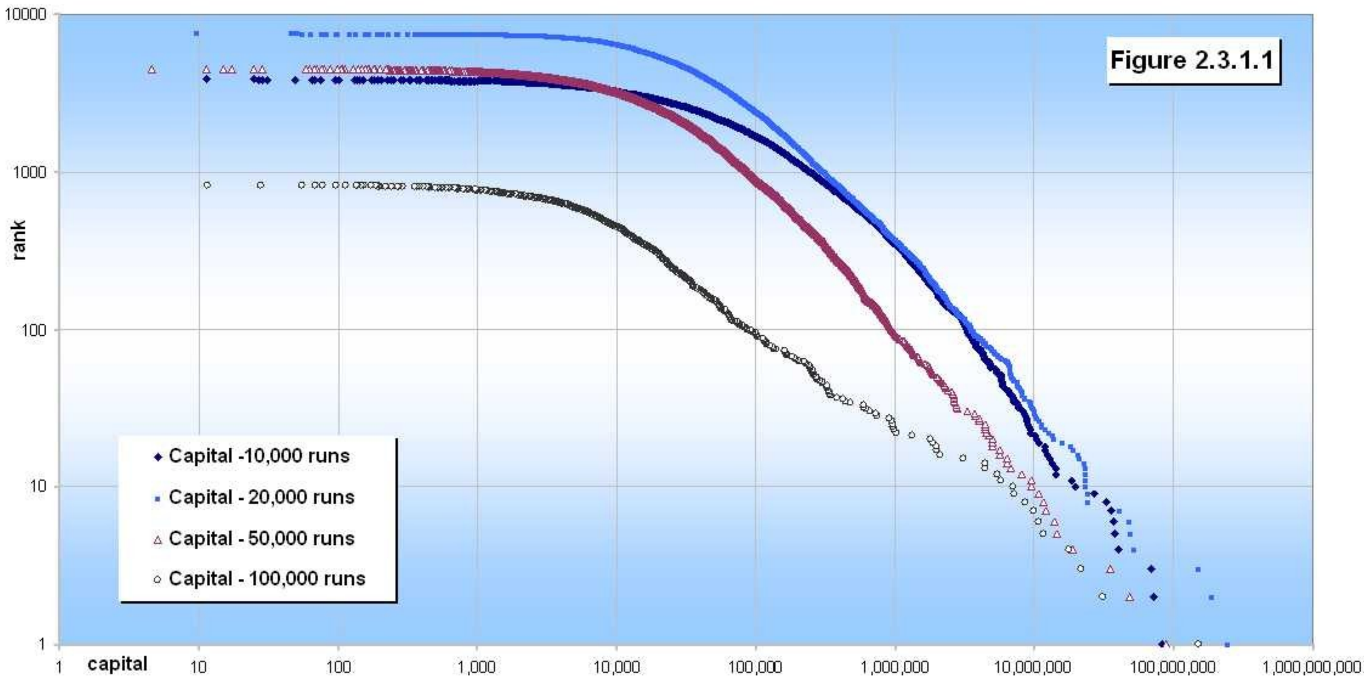


Figure 2.3.1.1
rank
capital
Capital -10,000 runs
Capital - 20,000 runs
Capital - 50,000 runs
Capital - 100,000 runs
10000
1000
100
10
1
1
10
100
1,000
10,000
100,000
1,000,000
10,000,000
100,000,000
1,000,000,000

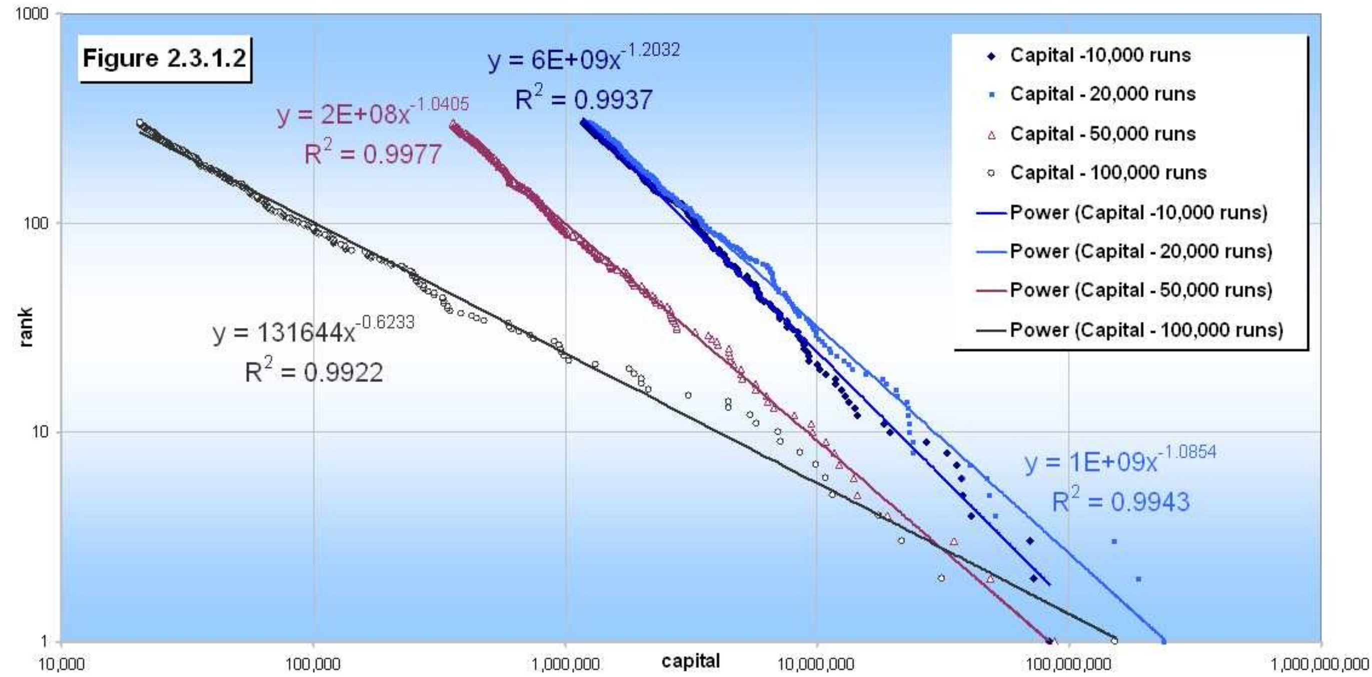


Figure 2.3.1.2
y = 6E+09x^-1.2032
R^2 = 0.9937
y = 2E+08x^-1.0405
R^2 = 0.9977
y = 131644x^-0.6233
R^2 = 0.9922
y = 1E+09x^-1.0854
R^2 = 0.9943
Capital -10,000 runs
Capital - 20,000 runs
Capital - 50,000 runs
Capital - 100,000 runs
Power (Capital -10,000 runs)
Power (Capital - 20,000 runs)
Power (Capital - 50,000 runs)
Power (Capital - 100,000 runs)
rank
capital
1000
100
10
1
10,000
100,000
1,000,000
10,000,000
100,000,000
1,000,000,000

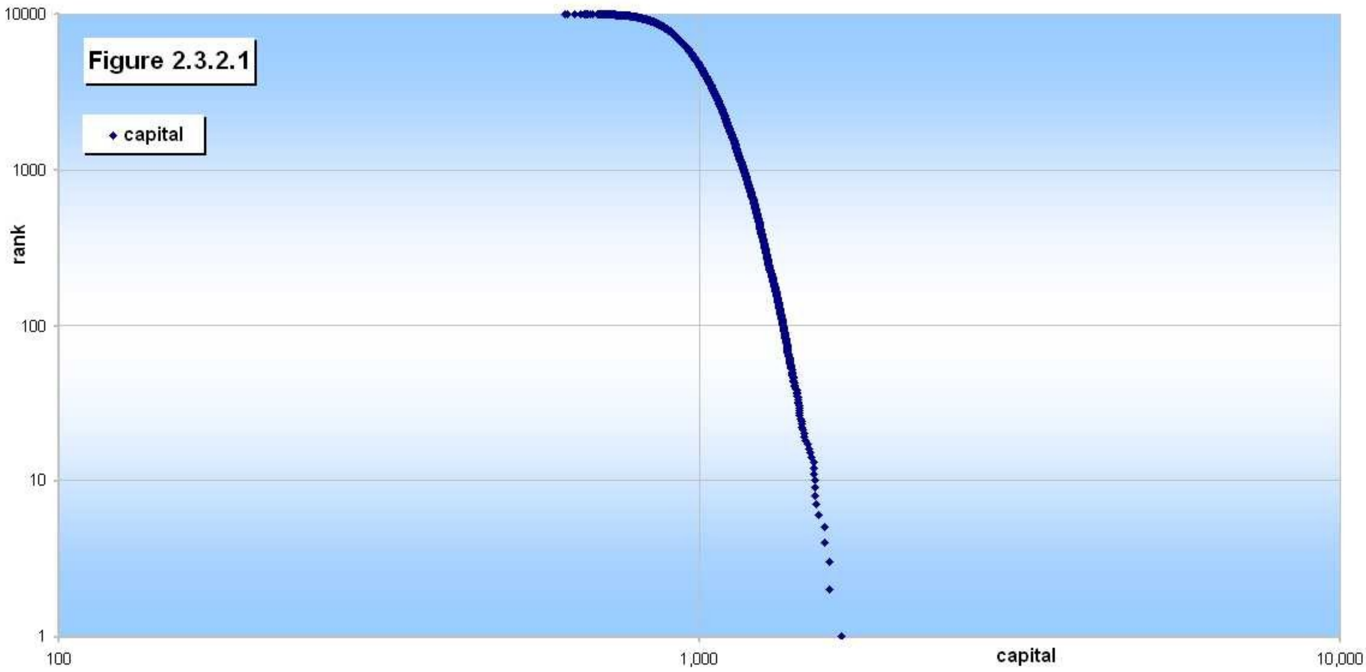


Figure 2.3.2.1
capital
rank
capital
10000
1000
100
10
1
100
1,000
10,000

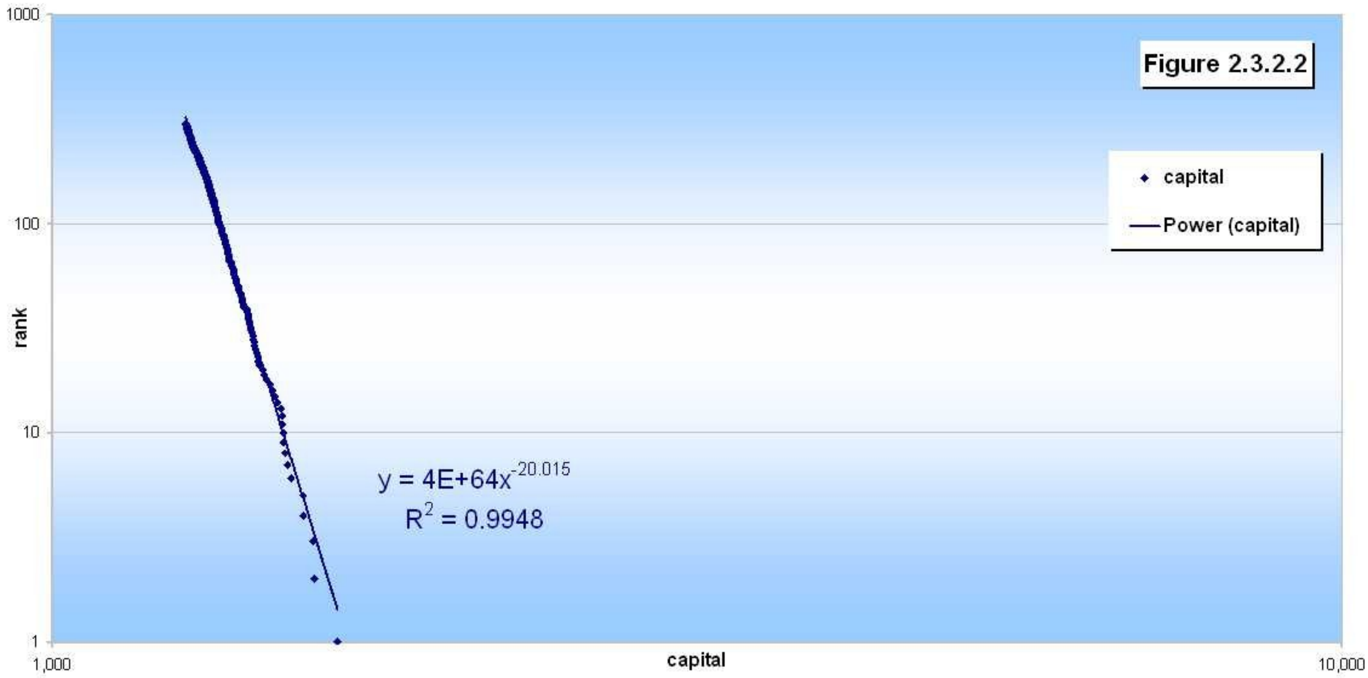


Figure 2.3.2.2
capital
Power (capital)
rank
capital
$y = 4E+64x^{-20.015}$
$R^2 = 0.9948$
1000
100
10
1
1,000
10,000

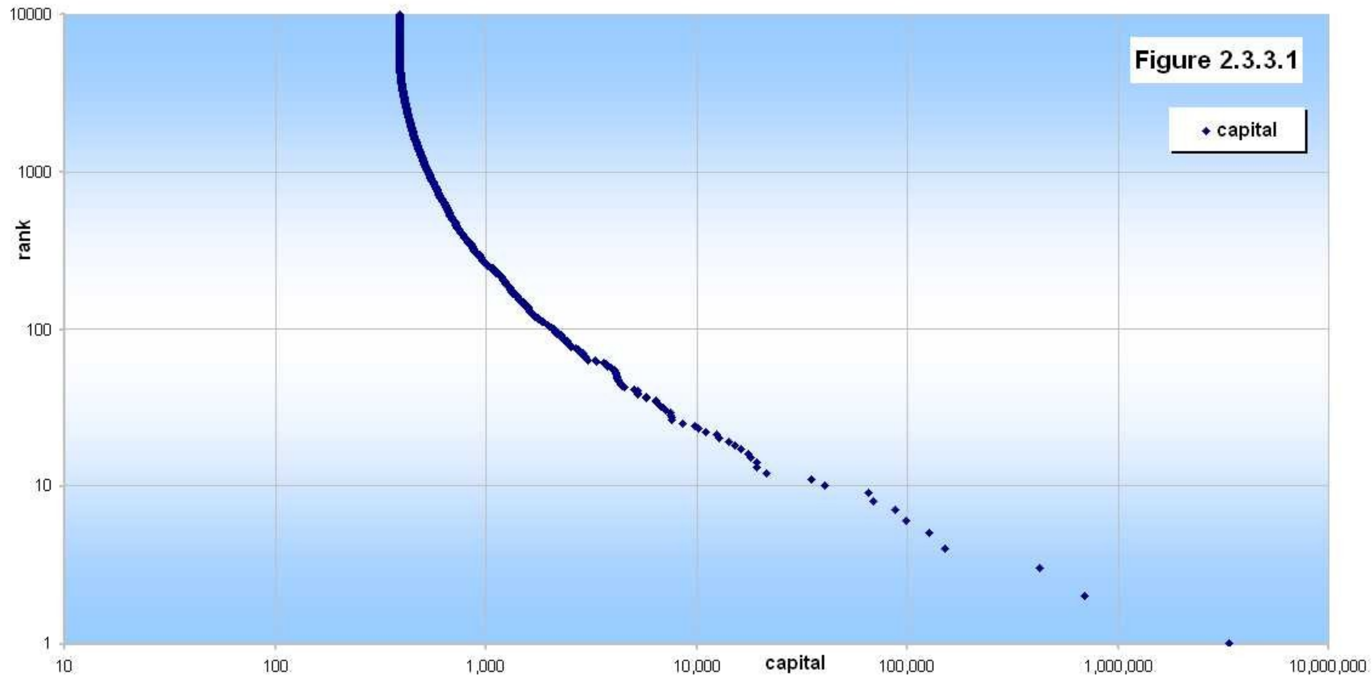


Figure 2.3.3.1
capital
rank
capital
10000
1000
100
10
1
10
100
1,000
10,000
100,000
1,000,000
10,000,000

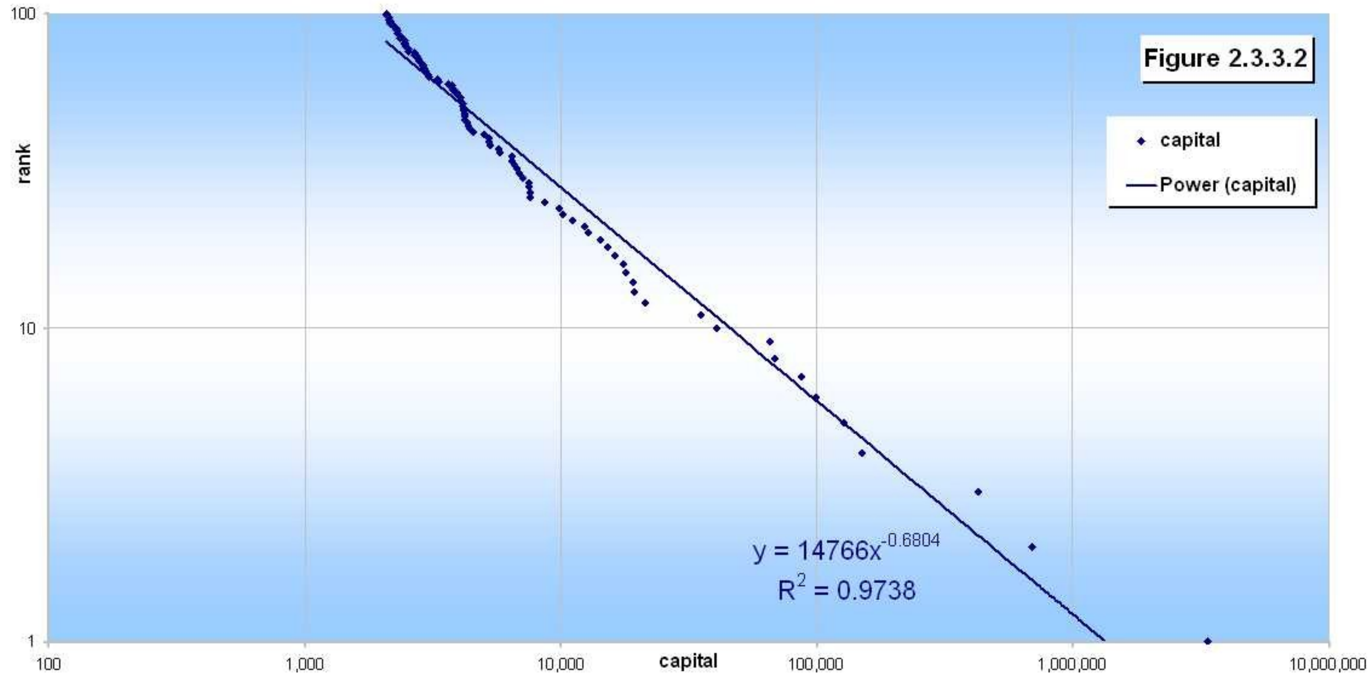


Figure 2.3.3.2
capital
Power (capital)
rank
capital
y = 14766x^-0.6804
R^2 = 0.9738
100
10
1
100
1,000
10,000
100,000
1,000,000
10,000,000

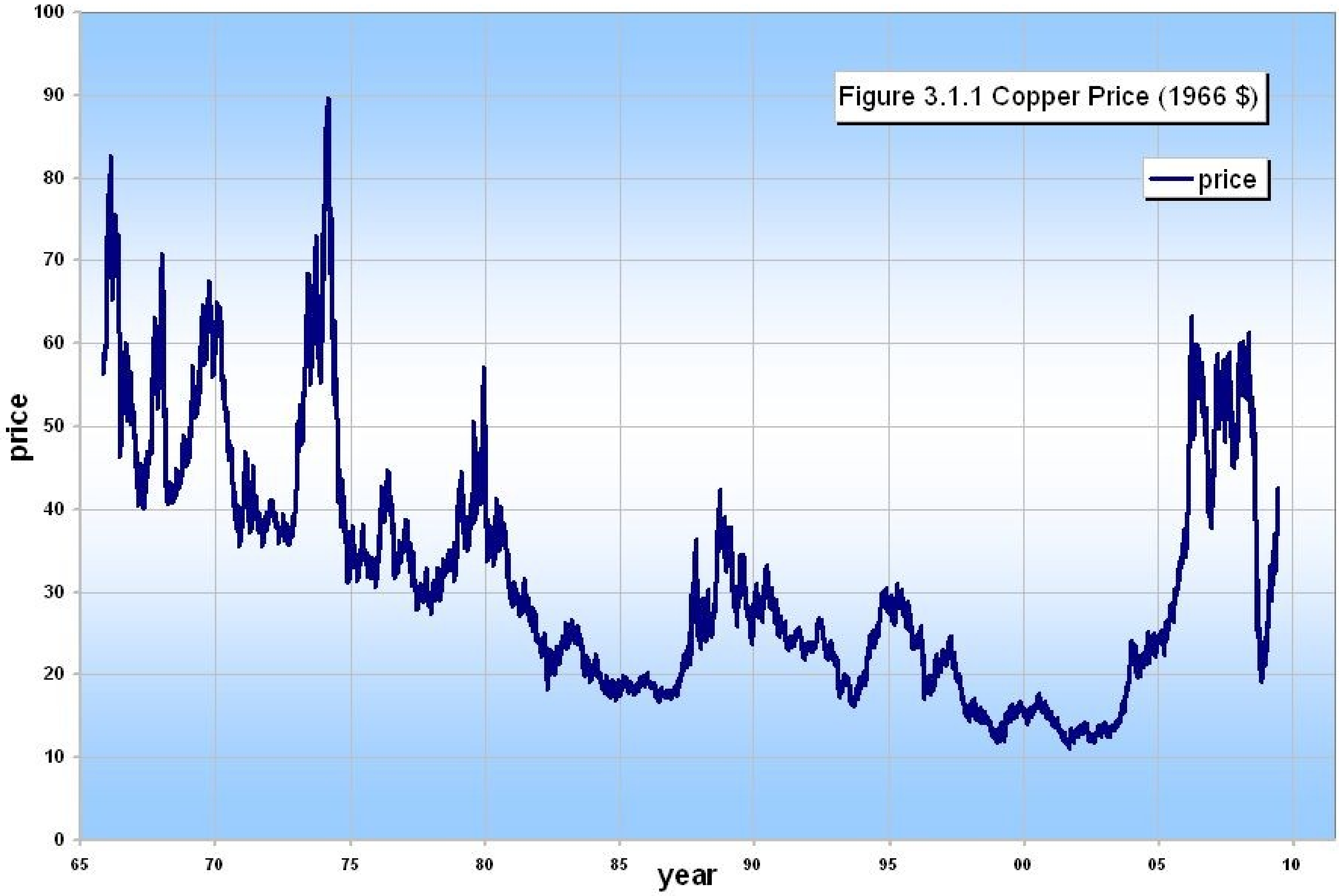


Figure 3.1.1 Copper Price (1966 $)
price
price
year
100
90
80
70
60
50
40
30
20
10
0
65
70
75
80
85
90
95
00
05
10


Figure 3.2.1 Copper Supply Curve
price
price
production
3
2.5
2
1.5
1
0.5
0
70
80
90
100
110
120
130

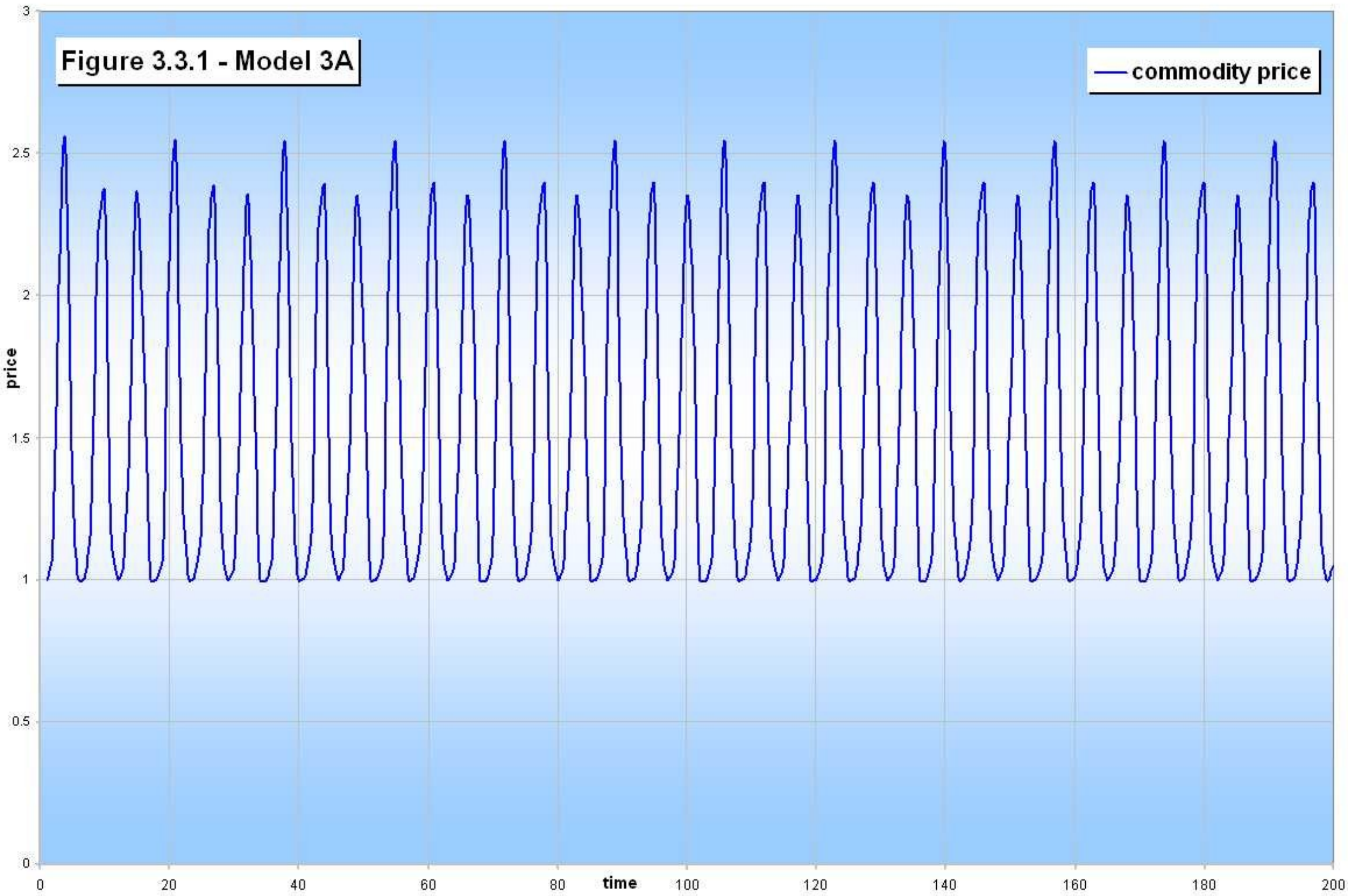


Figure 3.3.1 - Model 3A
commodity price
price
time
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
120
140
160
180
200

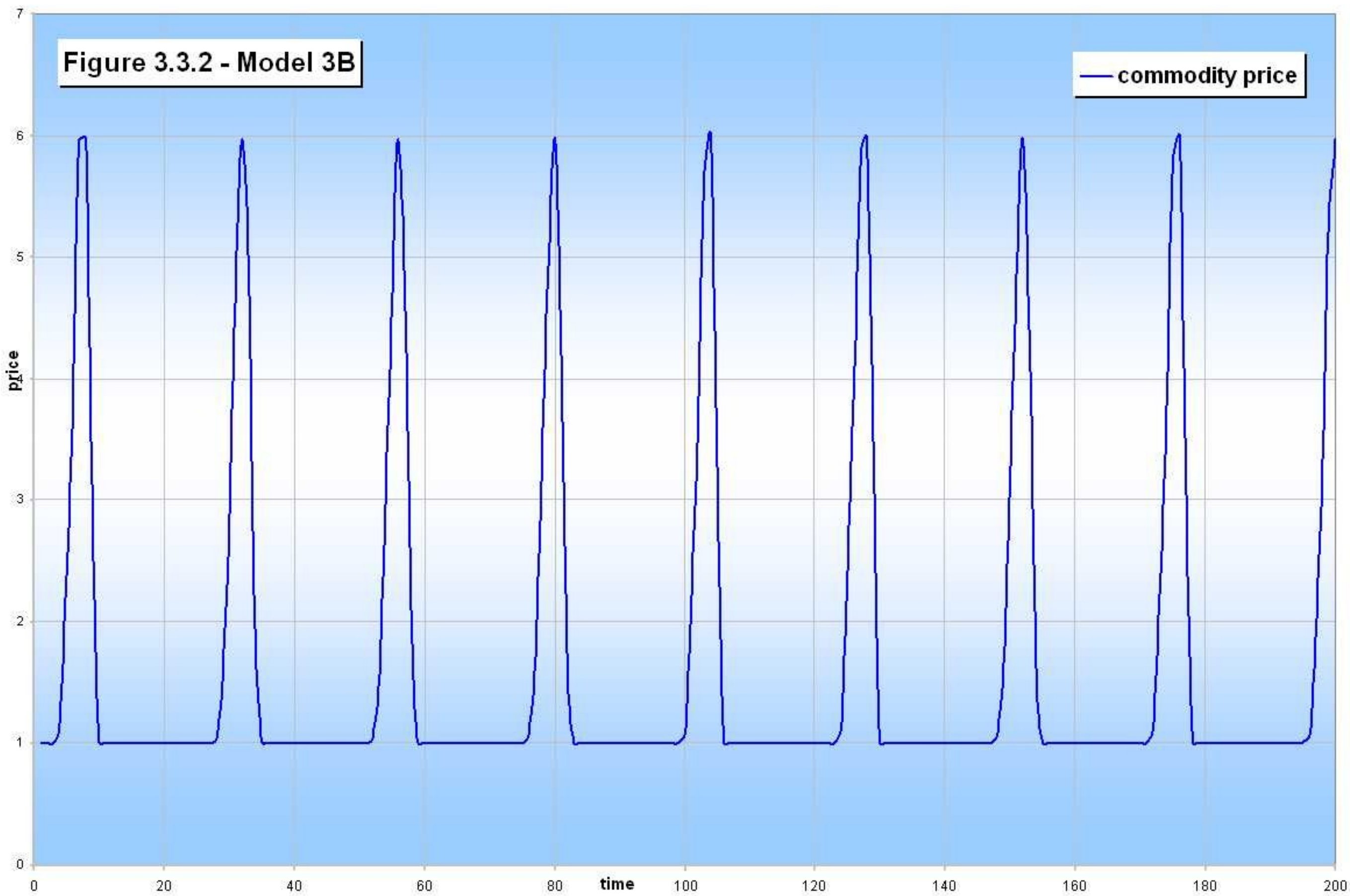


Figure 3.3.2 - Model 3B
commodity price
price
time
7
6
5
4
3
2
1
0
0
20
40
60
80
100
120
140
160
180
200

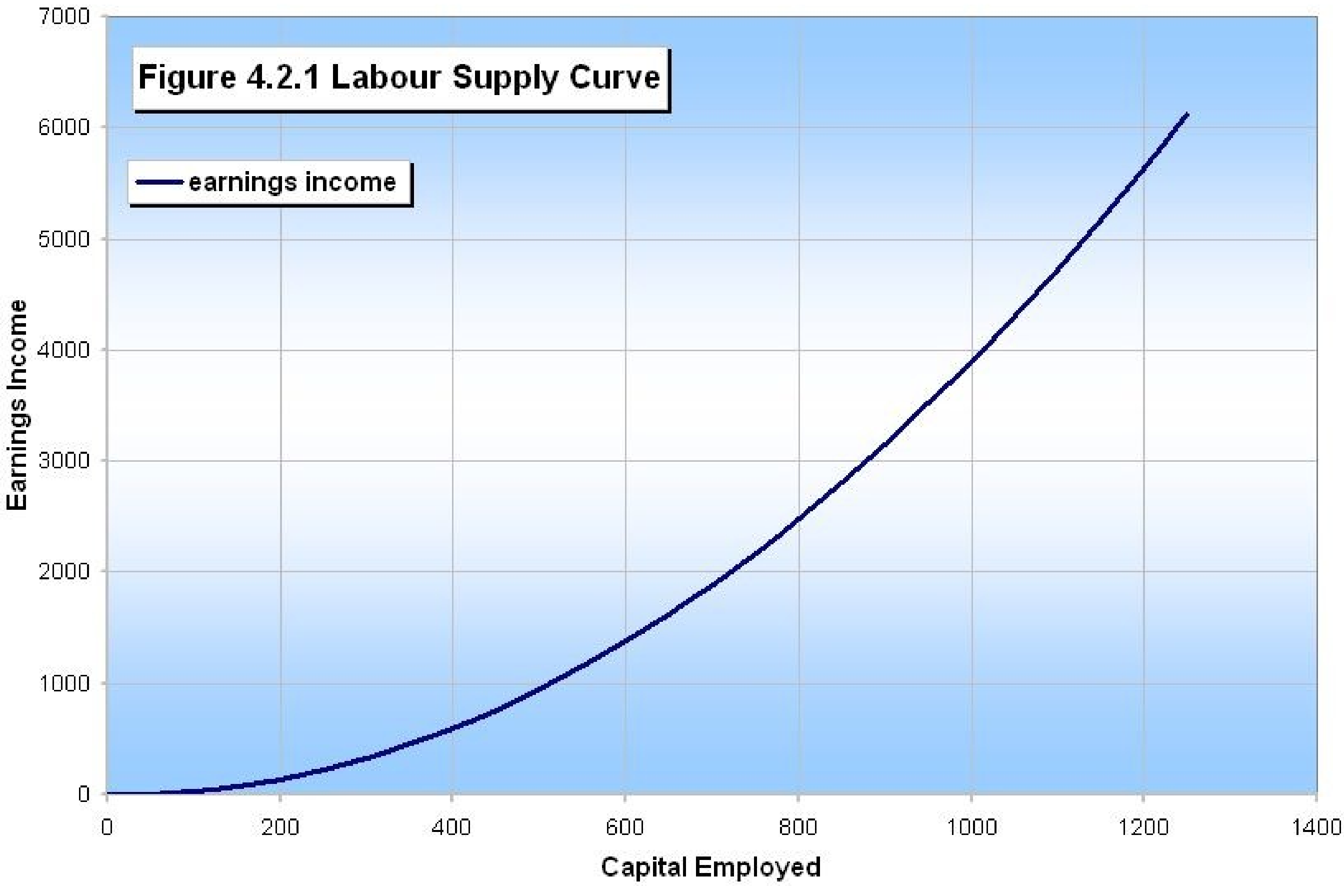


Figure 4.2.1 Labour Supply Curve
earnings income
Earnings Income
Capital Employed
7000
6000
5000
4000
3000
2000
1000
0
0
200
400
600
800
1000
1200
1400

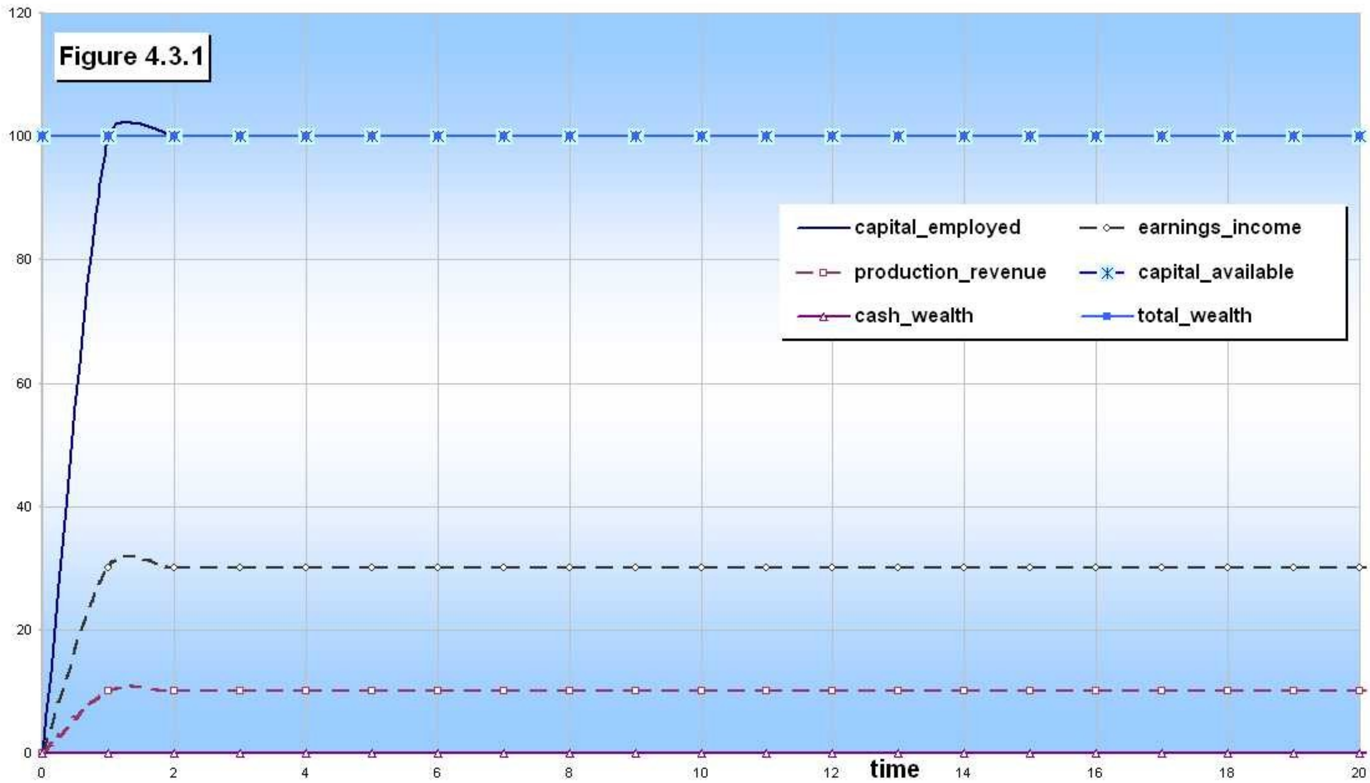


Figure 4.3.1
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth
time
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
20

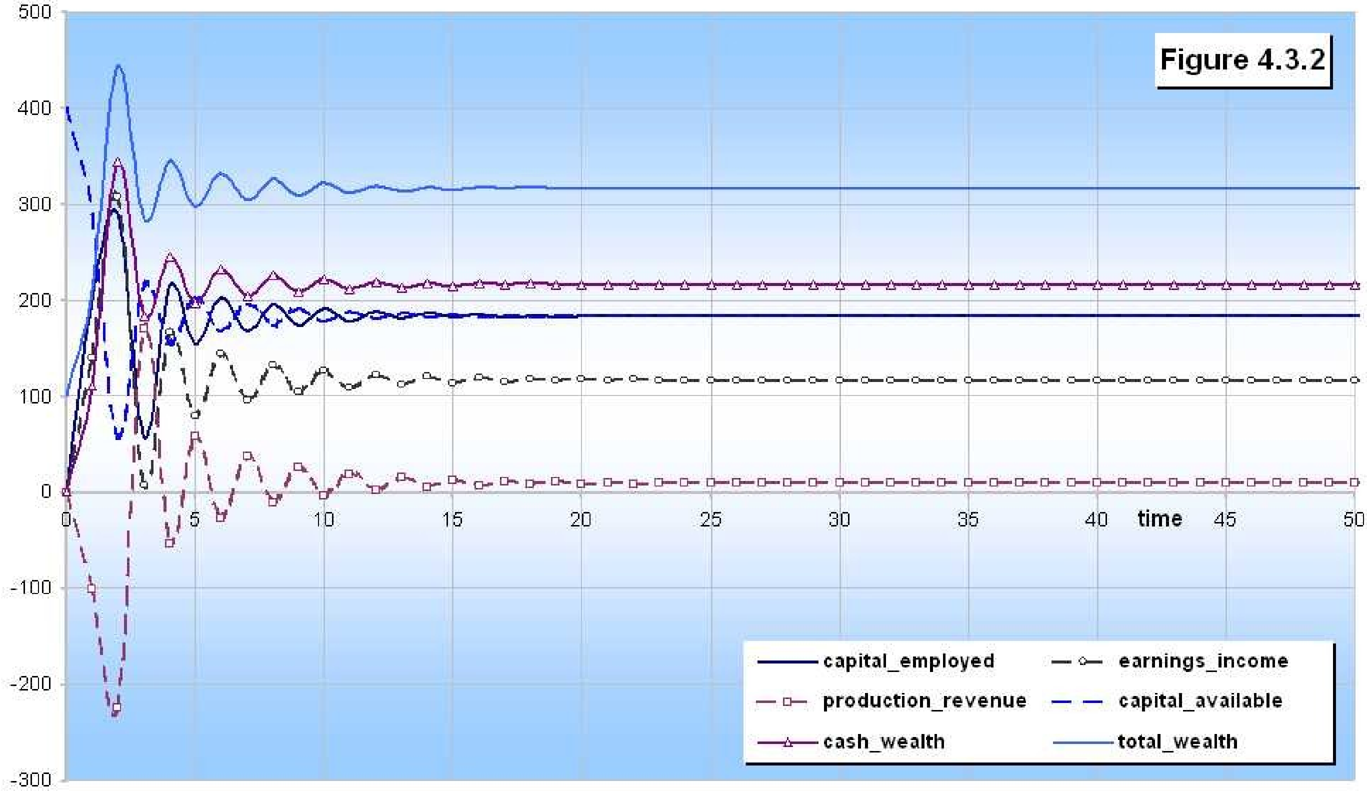


Figure 4.3.2
500
400
300
200
100
0
-100
-200
-300
0
5
10
15
20
25
30
35
40
time
45
50
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth

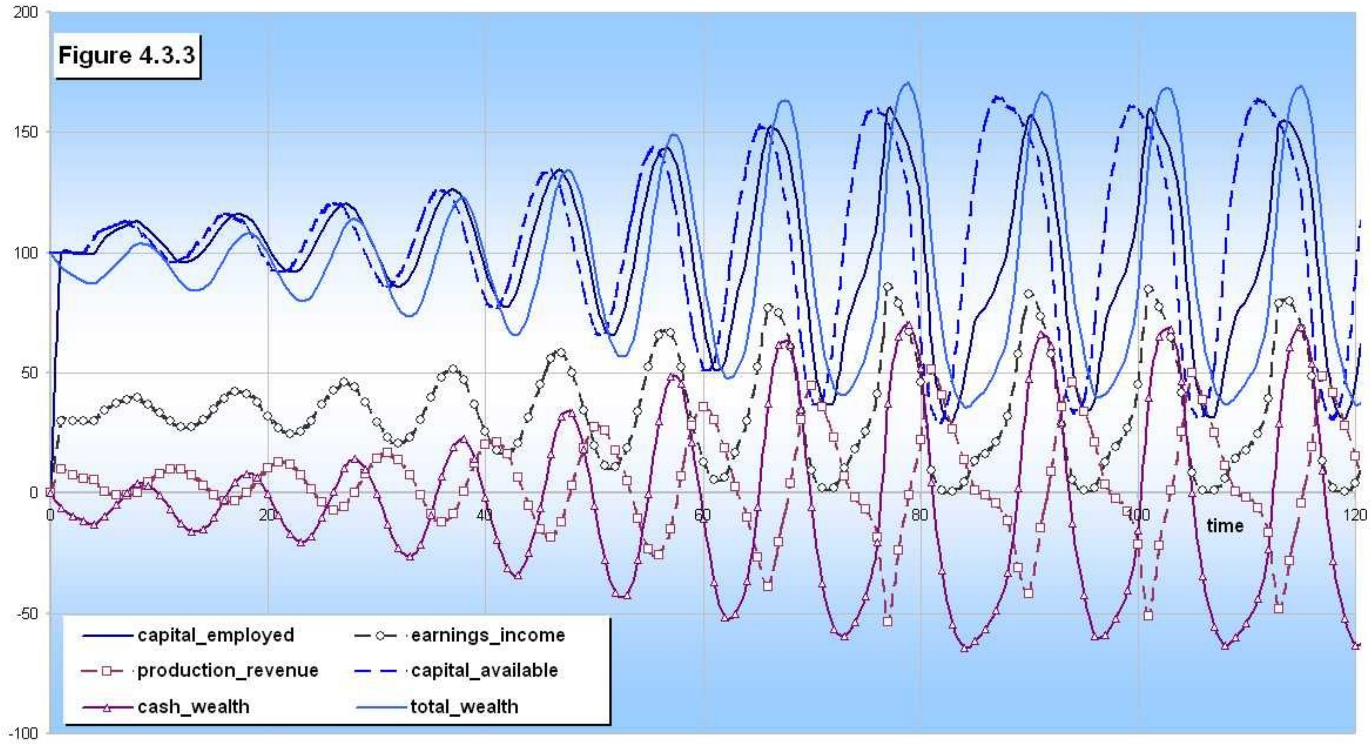


Figure 4.3.3
200
150
100
50
0
-50
-100
0
20
40
60
80
100
time
120
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth

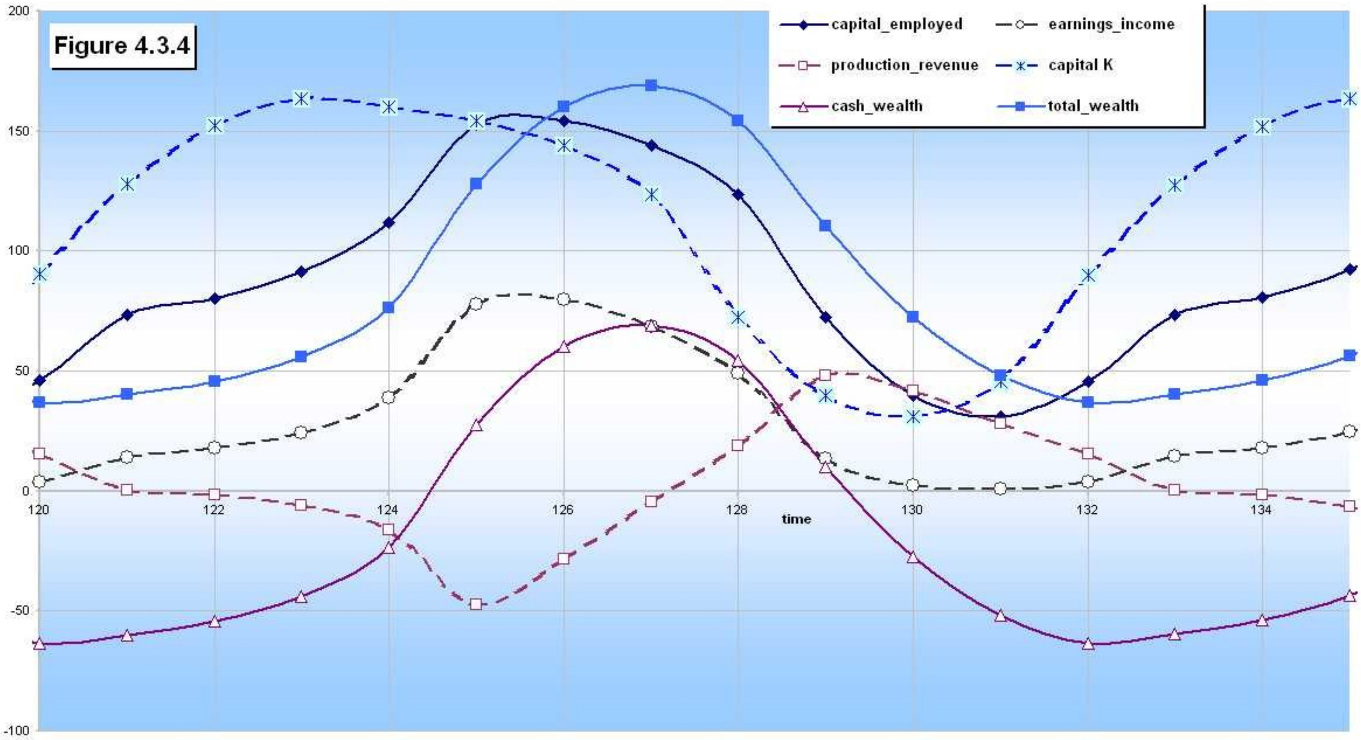


Figure 4.3.4
capital_employed
earnings_income
production_revenue
capital K
cash_wealth
total_wealth
200
150
100
50
0
-50
-100
120
122
124
126
128
130
132
134
time

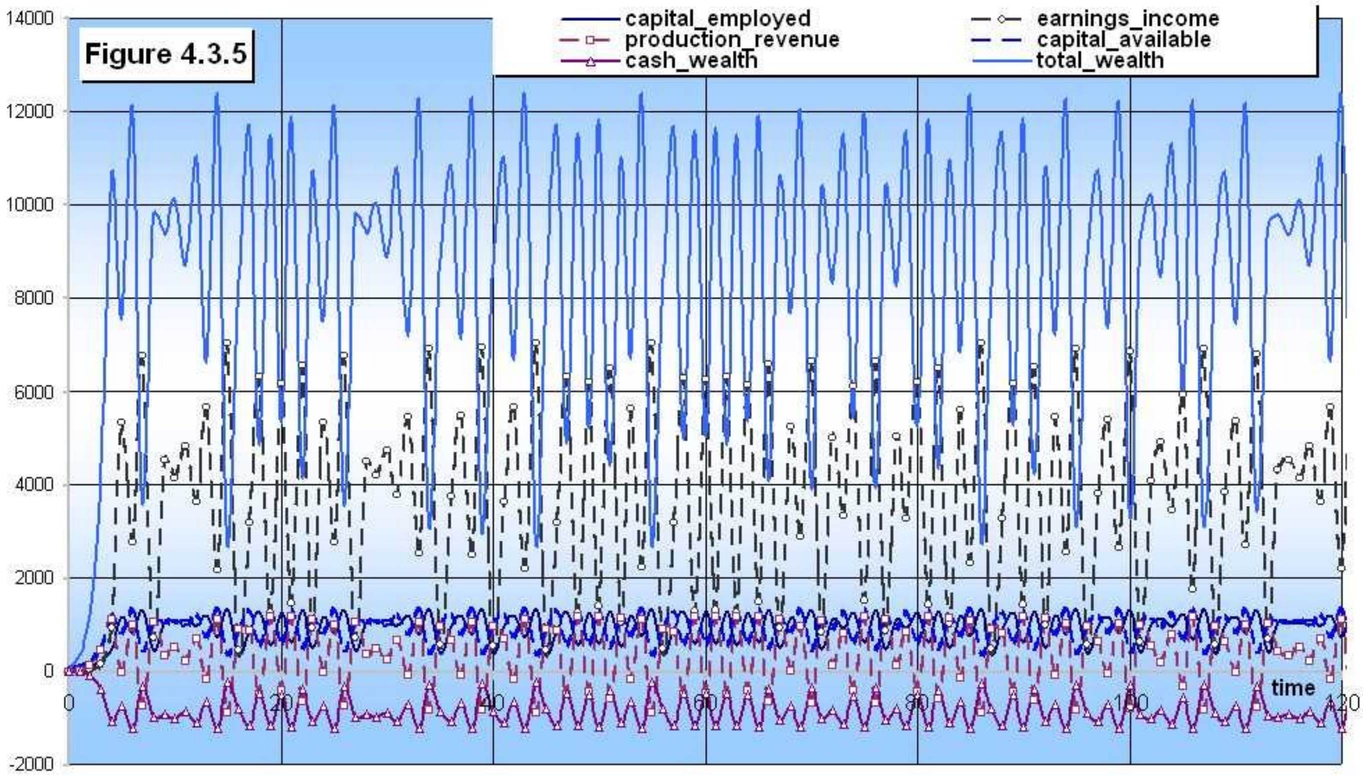


Figure 4.3.5
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth
14000
12000
10000
8000
6000
4000
2000
0
-2000
0
20
40
60
80
100
120
time

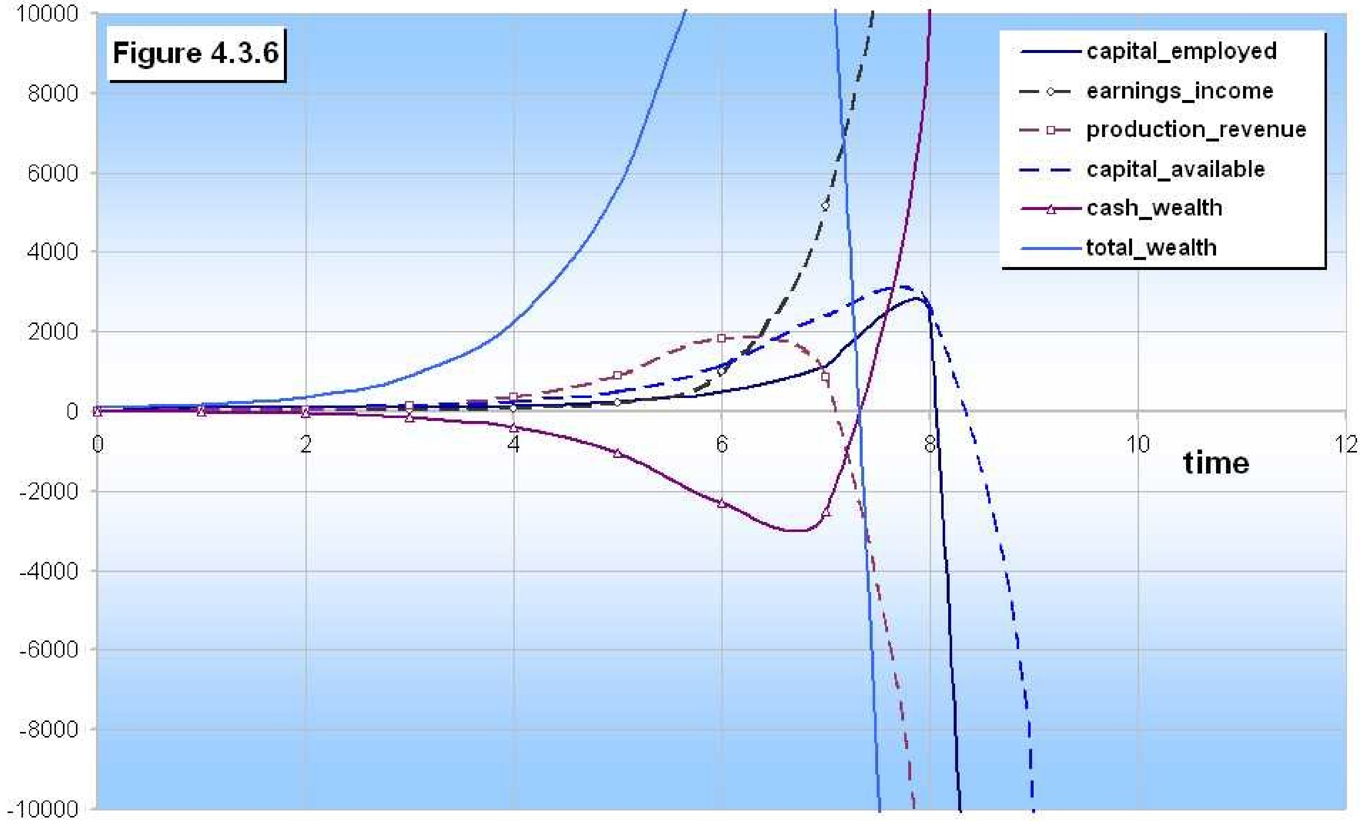


Figure 4.3.6
capital_employed
earnings_income
production_revenue
capital_available
cash_wealth
total_wealth
time
10000
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
0
2
4
6
8
10
12